OPERATION RESEARCH METHODS IN REGIONAL HEALTH POLICY

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INTRODUCTION

Regional policy of every country that is a member of a European Union have to take under consideration certain social aspects including health protection and social services. Health care standards, worked out due to many years of European Community existence, became obviousness for the society of the old members. Although the organization of health care in those countries is criticised by theirs citizens, it become an element of envy for the inhabitants of the countries that aspire to be a member of European Union. Among those people functions belief, that very soon after joining European structures changes for better will occur, at least the ones that will near them to expected standards.

Such expectations were also present in Poland. By many occasions it have been underlined in private conversations and in official viewpoint of central and local authorities representatives, that after joining European Community the great improvement in these matters will be possible.

It should be mentioned, that health care and social situation in post communist countries in the beginning of their transformation was not as bad as inefficient was their economy. Lack of success in field of efficient management and constrained personal liberty not always were followed by difficulties for citizens in everyday life. Common wastage was easily visible, but general care taken for proverbial worker was able to hide or cover many paradoxes in life. An example for well organized reaction for external thread was the case of disaster in Czarnobyl. Only Poland was able to deliver in just a couple of days so called Lugol liquid, that prevented damage of thyroid due to radiation, for the couple of million children. Such discipline and organizational skills were not shown at that time by any of Scandinavian and West European country.

Poland in the 90's went through a difficult period of social and health care structure transformation being at the same time under constant under subsidisation, permanent organizational changes and necessity of implementation of all tasks for "yesterday". In such conditions there should be more appreciation for courage and intransigent of authors who prepared the reform of health care services in the late 90's. The change of organization and the way of subsidisation of medical services became, despite later problems and difficulties, the proof for farsightedness of its authors and for willingness of preventing increasing complications and disputes.

The subject of interest of the author in the article is analysis of medical services management problems and presentation of some chosen optimization methods and decision techniques, which use in the regional social and health policy could streamline its functioning and help in handling inhabitants.

AREAS OF MATHEMATICAL METHODS APPLICATION IN MEDICAL SERVICES

Adaptation of operation research methods can have its place in many areas of medical services, both on the operational (regional) and strategic level. The basic task is to distinguish key dilemma in this sector and state the scale and vulnerability to the chosen methods. Not every element of analyzed sector is equally sensitive and vulnerable to their usage.

Classification o this elements on the operational level is strictly bound with the costs produced by medical services. Hence there can be discussion about following, more important elements of health care:

- closed treatment (hospital functioning),
- sanatorium treatment (rehabilitation stay),
- open treatment (infirmary, clinic),
- medicament distribution (pharmacy, sale without prescription),
- medical education (personnel education – physicians, nurses, life-savers, technical equipment operator),
- sanitary and epidemiology services activities,
- health care prophylaxis (inoculations, school hygiene, etc.).

The confrontation of functioning conditions in listed areas with real possibilities of use of statistical analysis and operation research techniques show some significant differences between them. Relatively easier is application of mathematical methods in hospitals, sanatoriums or by medicaments distribution, than it is in open treatment and medical education. In authors opinion the biggest possibilities of application emerge in closed treatment and in sanatoriums. On account of the significance for the local health policy the subject of main interest became closed treatment.

In polish financial reality of the medical care sector, the reasonable decision making on a regional level require between others consideration of verification of quantities of hospitals and hospital reference levels in region in reference to the public needs and real financial possibilities. So far experiences show, that there exist important difference between existing institutional structure and needs of population and financial means reserved for their handling. Thereby emerge conflict based on necessity of division of humble financial means between too many hospitals, that furthermore function in irrational way.

In the mentioned situation, it is advisable to use rational methods (meaning reasonable) of patient assignment to the network of hospitals. The ones, that in longer period of time will not find sufficient number of patients (ill), will have to be liquidated or completely restructured. Fundamental question reduce to establishing, who will be the one that makes allocation, which was mentioned above. Under assumption, that most patients choose for themselves a place for treatment, it could be

concluded, that perceptive statistical analysis of ward bed usage in previous periods of time should lead to the decision about changes in functionality profile of a hospital.

Thereby there can be detected some clues about excess of certain types of hospital ward beds and about lack of the others. In such a case the role of the decision makers would be reduced to changing the structure of ward beds in particular parts of a hospital and in some cases to close-up of unneeded units.

Unfortunately such solutions, that make most sense from economical point of view and are socially legitimated, wakes resistance not only from both some patients and employees from restructured unit, but also from local authorities, that are interested in supporting deficient units on their region. Change of structure in hospital units often leads to necessity of dissemination of some of the people employed in the hospital and in other cases to need of re-qualification or permanent change of working conditions and payment. Very rarely these changes are positive financially.

The most common effect of disputes between hospital managers and its personnel concern leaving in unchanged form unit that is constantly ineffective, generates debts and can't change hitherto existing policy.

The question, that is most often asked at that point is: is it possible to control number and reference level of dependent units in such way, that it enables constant improvement of their economical situation and at the same time will not provoke locally any social conflicts. It seems, that in such cases there is no other way then put unpopular but economically necessary reforms across.

Sustaining in artificial way constantly unprofitable or unnecessary units results in waste of social financial means and objectively deterioration of access to health care services for many potential patients.

Why then, in such a case very rarely come to important changes despite visible for everyone reasons?

One of the most important reasons is lack of distinct and understandable for everyone premises, that sustaining status quo is unprofitable for all people. As long as patients will not know, what they can expect in return for paid health care premium, they will not be interested in purposes on which their money are spent. Settlement of guaranteed medical services basket in return for monthly paid premium and affiliation of premium amount to the number and level of services is the best way to persuade majority of population to take interest in the information if their money are well spent or invested.

Similar problems to the ones mentioned above generate the management of rehabilitation facilities (sanatoriums, vacation and therapeutic centers, rehabilitations hospitals, etc.). The most important is to decide, to whom belongs analyzed facility- it concludes, which policy can be lead by its owner. The facilities, which are completely private are used entirely in commercial way, so their functionality is not different from normal firms preset for gaining profits. In case where sanatorium belongs to local authorities, usually their functionality can described as partially commercial – with some preferences for the inhabitants of the region. The biggest problem is with facilities, that belongs to the state. Usually they are underinvested and their functionality quite often unprofitable leads to the indebtedness, low quality of service and in conclusion to bankruptcy despite great value for the inhabitants and long tradition.

Exit from deadlock situation in such cases fetch to change of status of this units through full privatization or creation of possibility for the local authorities to takeover. This is the only chance to save well-deserved for medicine facilities with traditions sometimes up to several tens of years.

CHOICE OF TARGETS AND FUNCTIONALITY EFFECIVENESS OF MEDICAL SERVICES UNITS

Valuation of functionality effectiveness of health care services units sector can be made in similar way as it is done regarding firms and companies from the area of production or services. Health care is nothing else as certain service, so some decision techniques can be applied with confidence, that specifications of the area will be kept.

According to the authors beliefs the total valuation of effectiveness is based on two fundamental elements:

- working efficiency understood as relation (or difference) of obtained effects to bore efforts in established amount of time,
- work effectiveness understood as a degree of carried out tasks (targets), from the ones that were accepted at the beginning, also in established amount of time.

First element described as efficiency, rather do not have relation to decision making process and is treated as subject of economical analysis, to be more precise econometric. The base for the study of efficiency are stochastic cause-description models that enables objective valuation of changes scale that took place in studied amount of time.

Effectiveness valuation of carried out tasks can be achieved with use of various models for economical decision making. It is thought to make a valuation of a carried out tasks degree from the ones accepted in the beginning. Tools for this valuation can be found in various areas of operations research and mathematical programming.

Treating working efficiency and effectiveness in achieving tasks as two parts of functionality effectiveness of economical units it is assumed at the same time, that there exist a way of measuring indications of effectiveness in economy practice. For this purpose, valuation of activity effectiveness degree have to be preceded by previous specification and definition of set of targets, that have been given to a managers of the company by its owner. Therefore, before presentation of methods and models, which can be used for valuation of a degree of assumed to achieve tasks, there should be disputed their internal structure and dependence of carried out tasks character from specification of instantiated economy system, and also various property forms of medical services facilities[1].

Fundamental task of a company in classical market economy is achievement of the goals of its owner, which fetch to proving profitability, resistance for slump and ability to survive and develop regardless to the situation in the environment[2].

Basic issue by the try of functionality effectiveness valuation of studied unit is consideration, which decision criteria should be in force in the conditions of market economy. Criteria identification is heavily dependent on the kind of institution that we deal with – state-owned, communal or private. But if the certain company and its owner accept basic assumptions and market demands, than in the most important matters the difference will not be that big. The tasks carried out by private health care centres are the ones, which are the most market oriented due to the fact that

[1] Wide discussion in that area have been made by J. M. Dabrowski in the article [29].
[2] This is of course sort of simplification, which will be amplified later.

acceptance of the market principles is the base of their survival. The classification of tasks show beneath refer mostly to this group.

In this faze of deliberations we are not finally deciding, if while talking about full realization of the target, we think about gaining extreme point resulting from accordingly submitted character of criteria function, or if it concerns finding a set of possible to implement variants, which are resulting from constrain conditions and existence of economy environment. It seems that in complicated economy reality more important role should be attributed to the second approach.

The presented beneath classification of targets takes in to consideration their character, related with the type of company regardless from the environment in which it functions. In this context all targets are divided on productive, scientifically-technical, social and ecological. The division mentioned above is not fully useful to economy effectiveness analysis in hospital and others units of health care sector.

Due to a role of time and task realization period the targets can be divided in to the current, long term and constant.

In literature specialised in management there can be found a view, that considering significance and global extent it can be talked about following targets[3]:

- mission,
- strategic targets,
- tactic targets,
- operational targets.

Classification made in this way distinguish the significance of particular leadership actions depending on the management level. Under mission concept we understand the most important target, which for the hospital can be granting help on the highest possible level according to attributed reference level. Mission results from essence of unit function and can not be questioned by its leadership.

Strategic targets concentrates on global problems of studied unit. They determine tasks and undertakings field of the lower row for example costs optimisation and high level of services assurance. On the middle level emerge the tactic targets, which can be reduced to performance to the activities that assure realization of strategic target, for example a cut down of awaiting time for examination or operation. Operational targets refer to the lowest level and can be concerned to a smaller improvements, lead in the particular moment and assuring conditions success of the targets of a higher level.

Classification from the economist point of view concerns the choice of tasks, that should be posed before health care units, which implement typical tasks for the market economy.

They can be divided in to two groups[4]:

- bound with profitability category,
- bound with the risk level accepted in the company.

Among the targets bound to the profitability we distinguish:

- maximalization of earned accumulation or profit,
- sale maximalization of produced services,
- gaining “satisfactory” profit level,
- gaining the assumed percentage in sale on the medical services market.

In the group of targets referred to the risk, there are:

- survival of a hospital,

[3] Por.[Zarz UJ] p.89-90

[4] Compare work [145], p. 4-6

- maintain employment level,
- stabilization of income.

Targets mentioned above are off course only the part of the formulas from among many others with different extent and character. These targets emphasise various of views of an economist on the economical units, both distinct (amount of profit or income) and global (survival of a company). They combine marketing (market share), financial and social (employment) nature liabilities.

It is a matter of discussion if and how can matter of choice of so presented targets be referred to the reality of health care sector in the particular region. Division on mentioned in the beginning categories – according to the profitability and risk level – does not entirely match to the character of health care sector functions. If the hospital functions on the same conditions as other companies, than with confidence it has to take in to consideration profitability of conducted activities. It is like that, when the hospital is a commercial unit, and then the choice of target made by its owner is not that complicated. Mostly the hospitals and other similar units are managed by local authorities, and in such a case it is not that simple to state a distinct target.

Profitability in a longer period of time assure survival of a unit, stability and means for further development. In Polish conditions it is not that common phenomenon. Therefore it can be acknowledged, that the real target, that has to be implement by average hospital in Poland, is management of accumulated means in such a way, that they do not generate any debts. Task for the leadership would be to look for lacked founds for functioning in additional paid services, economical and reasonable property management, rationalization of employment and premeditated and well calculated investments. From these remarks clearly can be concluded, that those tasks can not be implemented by physicians but rather well prepared managers. Therefore any defection from rational economical policy as an effect of superior authorities intervention can take place under condition, that those authorities will cover eventual loses, that came from their decisions.

Talking about risk in case of medical services units we think about continuation of previous activities. Taking into consideration well being of patients, in the first place we have to refer to the targets combined with risks. In case of a small hospital, in a small city far away, its survival is a common goal not only for the employees and leadership, but mainly of potential patients, who practically do not have a possibility of choice in the matter of hospital care. In such a case very important is also maintaining employee structure, which in case of a small units and lack of external concurrence results in striving to maintain status quo as long as possible, despite clear signs to put restructuring process across.

Talking about gradation and task realization for regional health care program the author beliefs that there should be assumed following sequence of reasoning and actions:

- well being of patients from local society,
- real financial possibilities of functioning and sustaining hospital with specified reference level,
- well being of a stuff employed in hospital and real possibilities of changes in employment,
- rationalization of access to the stationary health care locally and globally.

Discussion carried out so far leads to the clear conclusion, that choice of the adequate target needs multi-criteria approach. In such specific area there is no chance for any criteria to become dominant and exclude others. Properties of multi-criteria tasks solution lead to a conclusion, that there is no chance of achieving results close to optimal. There is only possibility of finding compromise solution, which are no that easy considering the specific of the problem.

The most important problem stay the resolving target conflict. In the situation where there exist many financial and organisational constrains emerge conflicts, from which most typical are:

- if the unit should achieve financial stability in a long or in a short period of time,
- what is more important – achieving a profit, current consumption (earnings rise), or maybe investing to assure better concurrence position,
- if the main focus should be on development (new medical procedures, new services) or stabilizing achieved so far position,
- what is more important – gaining profit or implementation of social goals.

Focusing most of others on economical effects makes impossible to valuate essential "soft" target of an functioning Health Care Institution (ZOZ – pol. Zaklad Opieki Zdrowotnej) – implementation of social needs. Though it requires separate attention and is not vulnerable to the use of mathematical methods.

METHODS OF SOLVING MULTI-CRITERIA DECISION TASKS

For tasks of mathematical programming with many criteria functions, similarly to the majority of the non-linear programming tasks, there doesn't exist distinct universal method of solving this type of issues. Therefore important stage becomes search of the possible ways of finding satisfactory solution.

Such theme has been acknowledged as important and needed in case of valuation of ZOZ functioning. According to the author its importance comes from two reasons:

- prepared decision making tasks have mostly multi-criteria character, which is a result of necessary implementation not only one goal, but whole group with various complement,
- criteria functions most often are non-linear, so it require the knowledge of special methods for solving such decision making tasks .

While searching for the way of solving decision making tasks for valuation of activity effectiveness we have to remember, that possibilities of their use depend on the mathematical character of criteria functions, number of criteria, knowledge of descriptive parameters of the task, and also global size of the decision making problem. Although multi-criteria mathematical programming is conceptually important generalization of single-criteria programming, most often numeric methods of its solution are reduced to algorithms prepared for single-criteria optimization[5]. Most often it comes to bringing the task to the form, that enables usage of mentioned numeric methods. Basic problems related to the search of effective solution of a chosen decision making tasks depending on the compiled form

[5] Por. remarks in this area in work [47], p. 8

and number of criteria is shown beneath. These are only exemplary solutions. The full review of them would greatly oversize the article and would need extensive monograph.

Decision making criteria in economical issues often becomes the form of quotient. The idea of use the quotient target function can be implemented to the wider class of tasks with non-linear target functions.

For consideration of the finding effective solutions for this type of tasks issue let us assume, that proper decision making task has following form:

$$max\left[\frac{\sum_j c_j x_j}{\sum_j d_j x_j}\right], \quad j = 1,2,...,n \tag{1}$$

with constrains

$$\sum_j a_{ij} x_j \le b_i, \quad i = 1,2,...,m \tag{2}$$

$$x_j \ge 0, \quad j = 1,2,...,n \tag{3}$$

Lets say to simplify, that between constrains there is an assumption:

$$\sum_j d_j x_j > 0, \quad j = 1,2,...,n$$

Now we will define the set ***D*** of the acceptable solutions of outgoing task:

$$\mathbf{D} = \{\mathbf{x} : \sum_j a_{ij} x_j \le b_i \ , i = 1,2,...,m \ , \ x_j \ge 0\}$$

and also subset ***D(l)*** of the acceptable solutions, for which:

$$\sum_j d_j x_j = l = const., \quad j = 1,2,...,n$$

From this it concludes that constant λ is nonnegative number.

It can be proved, that [6]:

$$\mathbf{D} = \bigcup_{l \ge 0} \mathbf{D}(l)$$

and (by $\mathbf{x} \in \mathbf{D}$)

$$max\left[\frac{\sum_j c_j x_j}{\sum_j d_j x_j}\right] = max\left[\begin{array}{l} l^{-1} max \ \sum_j c_j x_j \\ x \in D(l) \end{array}\right]$$

for $l \ge 0$ i $j = 1,2,...,n$.

In such a case in reality there should be solved single-parameter task of linear programming:

$$max \ \sum_j c_j x_j \tag{4}$$

[6] Prove of this statement can be fund in the article [125]

with conditions

$$\sum_j a_{ij} x_j \le b_i, \quad i = 1,2,...,m \qquad (5)$$

$$\sum_j d_j x_j = l, \quad j = 1,2,...,n \qquad (6)$$

$$x_j \ge 0, l \ge 0, \quad j = 1,2,...,n \qquad (7)$$

and its solution is every time dependent from the value of the parameter l. If in the bottom of criteria function (l) are the costs bore on implementation of particular production plan, then the parameter l will be constrained from the top by real amount of that costs. At the same time there will be stated its variability interval. Solving the task (4) – (7) we will get vector $\mathbf{x}(l)$ such, that:

$$\sum_j c_j \hat{x}_j(l) = max \quad \sum_j c_j x_j$$

$$x_j \in \mathbf{D}(l)$$

$$j = 1,2,...,n$$

Vector $\mathbf{x}(l)$ and function $\mathbf{c}^T\mathbf{x}(l)$ are continous on their domain l and can be shown in following form[7]:

$$\hat{\mathbf{x}}(l) = \sum_j c_i(l)(u_i + l w_i)$$

$$i = 1,2,..., N-1$$

while :

$$N \in \left[1,2,..., \binom{m+n}{m+1}\right]$$

$$c_i(l) = \begin{cases} 1, & l \in (l_i, l_{i+1}) \\ 0, & l \ne l_i, (l_i, l_{i+1}) \end{cases}$$

and u_i , w_i are n-dimensional vectors, which are known together with l_i in support at solved parametric task like (4)-(7). Because search for optimal solution in the set $\boldsymbol{D}$ can be exchanged by search of optimum on their linear combination, written as $\cup\mathbf{x}(l_i)$ dla $i = 1,2,...,N$, so search for optimum (1) can be reduced to the examination of the function with only one variable. We will write it as follows:

$$l^{-1} \sum_j c_j \hat{x}_j(l) = \sum_i c_i(l) \left[l^{-1} \sum_i c_i u_i + \sum_i c_i w_i \right] \to max$$

$$i = 1,2,..., N-1$$

[7] Por. [125], s.95

Let $$\max_{l \geq 0} l^{-1} \sum_j c_j \hat{x}_j(l) = (\hat{l})^{-1} \sum_j c_j \hat{x}_j(\hat{l})$$

Vector $\mathbf{x}^* = \hat{x}_j(\hat{l})$ i san optima solution of an outgoing task, because

$$= \max_{l \geq 0} \left[\max_{x \in D(l)} \frac{\sum_j c_j x_j}{\sum_j d_j x_j} \right] = \max_{x \in D} \frac{\sum_j c_j x_j}{\sum_j d_j x_j}, \quad j = 1,2,\ldots,n$$

Thereby using parametric approach can be found solution of such issues, where constrained condition accompanies non-linear target functions in the form of quotient or product of linear forms, and as well when one of the linear forms is written as a power or root of another linear form.

Criteria function of a task that include at least couple of linear forms is mostly in practice a set that contains two kinds of criteria:

$$\max f_l(x) = \max \sum_j c_{lj} x_j \qquad (8)$$
$$l = 1,2,\ldots,s$$

where:

x_j – accordingly adjusted variables, and

$$\min f_l(x) = \min \sum_j c_{lj} x_j \qquad (9)$$
$$l = s+1,\ s+2,\ldots,k$$
$$j = 1,2,\ldots,n$$

For this reason, this decision making task is almost always conflicted, because there will not exist solution $\boldsymbol{x}$, that optimizes in set of acceptable solutions $\boldsymbol{D}$ all the target functions. Thereby there is no real possibility to determine optimal solutions by the assumption of criteria functions multiplicity. Consensual element stay then pursuit to achieve the smallest difference between optimum and value of particular target functions. Relative magnitude of deviation of such a target function from its optimal value will recognized as a measure of a comparative loses in realization of particular goal by the fixed solution.

There can also be assumed, that some of the criteria functions are to each other similar, so the optimal solution should be such, that it will minimize in the set $\boldsymbol{D}$ maximal (meaning - worse) deviations of particular target functions from their optimal variants in this set. This approach remind of reasoning taken from the basics of strategic games theory, namely choice of strategy of a player, that plays with nature the game with the sum of zero.

Effective solution of this task is reduced in the first part to finding k solutions of the smaller parts of the task, which comply constrains conditions together only

with the one of k criteria[8]. Optimal solution obtained in this way is signified by $\mathbf{x}_l^0$. Optimal values from particular target functions will be approximate:

$$F_l = f_l(\mathbf{x}_l^0) = \sum_j c_{lj} x_{lj}, \qquad l = 1,2,...,k \tag{10}$$

Every $\boldsymbol{x \in D}$ should comply inequalities:

$$f_l(\boldsymbol{x}) \leq F_l, \qquad l = 1,2,...,s \tag{11}$$

$$f_l(\boldsymbol{x}) \geq F_l, \qquad l = s+1,s+2,...,k \tag{12}$$

Second part of finding optimal solution depend on construction, and than finding optimal solution beside many goals. There must be found value of the function:

$$\min_{\mathbf{x} \in \mathbf{D}} \left[\max_{1 \leq l \leq k} \left\{ \frac{|F_l - f_l(\mathbf{x})|}{|F_l|} \right\} \right] \tag{13}$$

by settled conditions (3.1.11) - (3.1.12). Function that occur in curly bracket in (13) is not linear function of components of vector $\boldsymbol{x}$. Even though it is possible to prove, that in some areas it is linear and convex to the bottom. Than this function marked by x_{n+1}, has to comply to the combination of inequalities:

$$x_{n+1} \geq \frac{|F_l - f_l(x)|}{|F_l|}, \qquad l = 1,2,...,s \tag{14}$$

what, taking in to consideration (10), (11) and (12) allows to write it in the following form:

$$x_{n+1} \geq \frac{F_l}{|F_l|} - \frac{1}{|F_l|} \sum_j c_{lj} x_j, \qquad l = 1,2,...,s \tag{15}$$

$$j = 1,2,...,n$$

$$x_{n+1} \geq \frac{1}{|F_l|} \sum_j c_{lj} x_j - \frac{F_l}{|F_l|}, \qquad l = s+1, s+2, ..., k \tag{16}$$

$$j = 1,2,...,n$$

[8] Por. [110] i [137]

Inequalities (15) and (16) are linear toward uncertain variables x_j ($j = 1,2,...,n$) and x_{n+1} . Compliance of inequality (15) and (16) ensure, that $x_{n+1} = 0$. In this moment, there exist possibility of forming analogical task to (13), which is easier to solve because it is a task of linear programming:

$$L(x) = x_{n+1} \rightarrow \min \tag{17}$$

with conditions

$$\sum_j a_{ij} x_j = b_i, \quad i = 1,2,...,m \tag{18}$$

$$\sum_j r_{lj} x_j + x_{n+1} \geq R_l, \quad l = 1,2,...,s \tag{19}$$

$$\sum_j r_{lj} x_j - x_{n+1} \geq R_l, \quad l = s+1, s+2,...,k \tag{20}$$

$$x_j \quad ³ 0, \quad j = 1,2,...,n+1 \tag{21}$$

where:

$$r_{lj} = \frac{c_{lj}}{|F_l|} \quad , \quad R_l = \frac{F_l}{|F_l|} \ , \ l = 1,2,...,k \ , \quad j = 1,2,...,n$$

Task (17) - (21) can have one optimal solution. In such a case it is efficient considering processed target functions in a way, that through its changes, there is no possibility of decreasing relative deviations of the group of target functions, between others maximal deviation. But it does not exclude situation, in which there can be more optimal solutions. In such case the search for the efficient solution should be continued.

Finding optimal solutions with use of described above method is beneficent in case, when the optimization criteria of both target functions are the same (minimum or maximum). If in reality it is different, the target functions can't be treated equally, meaning it is almost impossible to take into account accepted in the beginning assumptions.

If the structure of criteria set is hierarchical, meaning that it is possible to put them in order by decreasing priority evaluating their importance and given to them by evaluative, then it would be advisable to use the method of putting criteria in sequence (hierarchical)[9]. It is assumed then, that particular criteria functions are not substitutable, so the change of the one of them does not have influence on the value of other. Compromise solution meaning accepted criteria order is optimal solution of the last from the set of *k* problems. According to that we optimize given problem at first by the first and most important criteria creating a set of solutions that fulfil its demands. Then we optimize again but this time by the second most important criteria, where the set of accepted solutions would be the set that we created by the first criteria optimization. An in this way we proceed until there is no criteria left.

[9] In such case it is assumed, that particular criteria functions are not substitutable, so the change of the one of them does not have influence on the value of other. This method has been presented between other in work [32]. It is also mentioned in the article of R. Slowinski [135].

The word “optimize” is not used literally in this context, but it means discrimination of certain sets, which can’t be in relation to this single point sets.

A possibility of finding solution gives approach that uses threshold constrains. There is no assumption about substitutive relations between criteria. Using it demand on the beginning designation for the particular criteria (without the first one) the threshold values P_l, that mean essential minimal but as well acceptable level of particular criteria. In this case the compromise solution can be found by solving the following task:

$$L(x) = f_1(\mathbf{x}) \rightarrow max \qquad (27)$$

with constrains

$$f_l(\mathbf{x}) \geq P, \quad l = 2,3,...,k \qquad (28)$$

Task (27-28) we will consider from the usability theory point of view. According to (28) particle usability for criteria f_l should be constant above the threshold value and “endlessly unbeneficial” under it. Usability function can be written as follows:

$$U_l(f_l) = constans \quad , \text{for} \quad f_l = P_l \qquad (29)$$

$$U_l(f_l) = -\infty, \quad \text{for } f_l < P_l, \qquad (30)$$

if $l = 2,3,...,k$.

In practice advisable is to make mentioned approach more flexible. Lets observe, that general flaw of the presented method stays the lack of differentiation of importance of particular criteria. In another words, every criteria is just as well as the other one if it comply with (29) and just as well bad if comply (30). In this situation very helpful would be less strict treatment of particular criteria and solving the task with different values and combinations P_l . It seems that more loose treatment of constrains P_l regarding to the particular criteria would effect in finding many acceptable solutions for the task (27 – 28). Thanks to this there would exist possibility of simultaneously consideration also the importance of particular criteria.

Supported usability functions can be also basis to build another type of decision making tasks. If the form of usability function is known, than the decision making task can be written as:

$$U[f(\mathbf{x})] \rightarrow \max \qquad (31)$$

with constrains

$$\mathbf{x} \in \mathbf{D} \qquad (32)$$

Usability function most often is shown in form of weighted sum:

$$U[f(\mathbf{x})] = \sum_{l=1}^{k} w_l f_l(\mathbf{x}) \qquad (33)$$

with values $w_l > 0$, $l = 1,2,...,k$ as weight factors. Function is additive and its elements are partial usability's:

$$u_l [f_l (\mathbf{x})] = w_l f_l (\mathbf{x}) , \qquad l = 1,2,...,k$$

Expression (33) is in this case real presentation of global decision maker preference model. Using the definition of substitution factors can be proved[10], that in case of weighted sum (33) and using by definition the substitution factors it is:

$$s_{lr} = \frac{\partial U(f)}{\partial f_l} : \frac{\partial (f)}{\partial f_r} = \frac{du_l}{df_l} : \frac{du_r}{df_r} = \frac{w_l}{w_r} \qquad (34)$$

Using correctly a global decision maker preferences model in form (31 – 33) require fulfillment of the following assumptions:

- stability of substitution factors in the whole variability area $f(\mathbf{x})$, pointed out in (34),
- mutual criteria independence pointing at preferences, meaning that particle usability of particular criteria is independent from particle usability of the rest of criteria

Method based on usability function gives effective solution, but practical accomplishment of conditions mentioned above and precise designation of weight factors brings out big difficulties. Not always it is possible, and most often very difficult to express decision maker preferences directly in the form of usability function. Therefore exist also another proposal of an approach to building and solving decision making model in global synthesis of decision maker preferences model a priori conditions.

Lets assume, that exist real information, which consists of target values of criteria, treated as points of reference. In such case the natural aim of the one solving the problem stays finding in $\boldsymbol{D}$ compromise solution, which picture in extent of criteria lays possibly closest to the chosen point of reference. To be more precise about expression "possibly closest" we'll define a measure to valuate distance in criteria extent, called distance function. As mentioned distance function can be taken norm L_p defined as follows:

$$L_p(\mathbf{x}) = \left\{ \sum_{l=1}^{k} \left[\frac{M_l - f_l(\mathbf{x})}{s_l} \right]^p \right\}^{p^{-1}} , \quad p = 1,2,...,\infty \qquad (35) -$$

while

$$M_l = \max_{\mathbf{x} \in \mathbf{D}} \{ f_l(\mathbf{x}) \}, \quad l = 1,2,,,,,k$$

and $s_l > 0$ are factors assuring possibility of comparison of particular elements of the sum. If all functions f_l are positive, then $s_l = M_l$, whereas $s_l = M_l - m_l$, if some f_l (*for* $l = 1,2,...,k$) can take positive and negative values in the set $\boldsymbol{D}$.
Values m_l are defined as follows:

[10] More abort that prove in por. [135].

$$m_l = \min_{\mathbf{x}\in\mathbf{D}}\{ f_l(\mathbf{x})\}, \quad l = 1,2,...,k .$$

Solution $\mathbf{x}^{(p)}$ is a compromise solution considering p, if

$$min\ \{ L_p(\mathbf{x})\} = L_p\ [\mathbf{x}^{(p)}].$$

Such solutions are efficient for $1 \le p < \infty$ and at least one is efficient for $p = \infty$. The higher is the value p, the higher participation in function L_p have components matching the highest particle deviation. In literature it is stressed[11], that the most important examples are the ones, for which $p = 1, 2$ or ∞.
For $p=1$

$$L_1(\mathbf{x}) = \sum_{l=1}^{k} \frac{M_l - f_l(\mathbf{x})}{s_l} \qquad (36)$$

that is, sum of a relative particle deviation.
If $p = 2$, then

$$L_2(\mathbf{x}) = \left\{ \sum_{l=1}^{k} \left[\frac{M_l - f_l(\mathbf{x})}{s_l} \right]^2 \right\}^{0.5} \qquad (37)$$

is a Euclidean distance of a ideal point from the point image $\boldsymbol{x}$ in criteria extent.
Finally, if $p = \infty$, then

$$L_\infty(\mathbf{x}) = max[\frac{M_l - f_l(\mathbf{x})}{s_l}], \qquad l = 1,2,...,k \qquad (38)$$

is maximal relative particle deviation.

Choice of the level of p should be preceded by perceptive economical analysis and interpretation of distance function L. Norms L_1 and L_∞ have also this advantages, that they reduce multi-criteria issue to the single-criteria one. It enables solving such a tasks in the standard way for example using the simplex method.

Most important postulate for the ones using in praxis methods of multi-criteria programming are:

- conception perspicuity and simplicity of implementation,
- guarantee of getting compromise solution, which is efficient,
- independence from the positive linear transformations of particular criteria functions,
- possibility of dialogue,
- small calculation expense (time of processor and amount of needed memory),
- good convergence,
- possibility of correction of previous decisions,]
- little vulnerability on the inconsistencies in decision maker answers,

[11] Por. further remarks in [128], [135] and [32]

- questioning of decision maker properly to their possibilities,
- no limitation in choice of compromise solution, meaning lack of suppression of the solutions to some local area or to the set of peak points by the amounts of acceptable solutions,
- small number of solution in the set shown once to the decision maker for valuation.

Some of this postulates are for an application in the effectiveness analysis less important (eg. possibility of dialogue), but their range prove that high requirements are posed before the presented methods.

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