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From research on mortality of old old people in EU countries of the Baltic Sea Region

Introduction

Together with extending length of people's life, more precise identification of regularities in mortality of elderly people becomes more and more important. More often, so called fourth age group (besides children, adults, elderly people) is singled out – the old old, that is people at the age of over 80 years. This caesura of age is accepted not only in demography, but also in researches within the scope of biology, gerontology and others. Changes in the age structure and intense ageing of population in different regions of the world brings with it significant consequences of social and economic kind. Therefore, there is nothing surprising in the fact of heightened interest in mortality of people in this age group, and hence in the possibilities of modelling proper functions describing the duration pattern for those people. Adapting models describing length of human life for the old old leads most frequently to overestimation of mortality, because over certain age there is a noticeable decrease in the rate of growth of probability of dying. Accepting such an assumption, of a decrease in the rate of growth of probability of dying of the old old, is connected with a selection intensifying with age, which causes that up to the old age live people of the best health. Changes are also visible in the relation between endo- and exogenous factors of deaths, that is „the risk of the background – environment" and the risk connected with the processes of ageing of an individual [3], [6], [8].

Basic question which arouses in when studying mortality of the old old regards the possibilities of adapting mortality models. The review of the models adapted in the analysis of mortality of the old old was presented most extensively in *The Force of Mortality at Ages 80 to 120* in joint authorship of A.R. Tchatcher, V. Kannisto and J.W. Vaupel and in *Modelowanie umieralnosci osób w wieku 80 lat i wiecej* by M. Gazinska, M. Mojsiewicz and J. Purczynski. Doesn't the need of searching new life duration functions for the old old exist, and shouldn't we seek better and more effective methods of estimating these models? Most certainly, yes.

On the basis of experience in modelling mortality of elderly people and the old old placed in [1], [2], [3], [4], [5], the main empirical aim of the paper was to carry out a comparative analysis of mortality of the old old in the EU countries of the Baltic Sea Region. All the empirical calculations have been based on data gathered in the database: *Kannisto-Thatcher Database on Old Age Mortality at the Max Planck Institute for Demographic Research*, concerning mortality of people aged 80 and more.

1. Life duration models for elderly people

Making assumptions that length of human's life is a continuous random variable, allows to define functions useful in analysing distribution of length of life, that is the life duration function and force of mortality function. Let T ($T = 0$) be random variable expressing time until death of a person belonging to a particular population in a moment of time $t = 0$. Let the probability that death occurs later than in a moment $t > 0$ or that a person in time $t > 0$ will be a part of a population (will live at least till the time t), be life duration function $S(t) = P(T > t)$, $S(0) = 1$, which is a monotonic and non-increasing function and $\lim_{t \to +\infty} S(t) = 0$. Cumulative distribution function (CDF) of random variable T, which is complementary to the life duration function $F(t) = 1 - S(t)$, describes the survival model. An important tool in survival analysis is the of force of mortality function $\mu(t)$[1]. This function expresses the probability of dying for short increments of time. Besides the assumptions and the property regarding each function separately, there are some mutual dependencies [7, p. 36].

In the models for discrete random variable T, life duration function $S(t)$ corresponds to the probability of duration one year by a person at the age of $x - p_x$, and survival function $F(t)$ to the probability of dying during a year of a person at the age of $x - q_x$.

The dependence between the force of mortality and the probability of dying is expressed by this formula:

$$q_x = 1 - e^{-\int_x^{x+1} m_t dt} \,. \qquad (1)$$

In practice, while estimating empirical values of force of mortality, rectangular method for numerical integration is used:

$$\int_x^{x+1} m_t dt \cong m_{x+1/2} \,, \qquad (2)$$

where: $m_{x+1/2}$ – force of mortality indicated in the middle of interval $\langle$ x, x+1 $\rangle$.

A useful dependence arises from formulae (1) and (2):

$$m_{x+1/2} = -\ln(1 - q_x) \,. \qquad (3)$$

The functions that has been singled out (CDF – survival function, duration function and force of mortality function) and the probability density function (PDF) related to time of life duration create so called pattern (model) of whole population's life duration. Models of life duration, including life duration model of elderly people can be separated into two basic groups: nonparametric models (life tables), for which analytical forms of functions describing life distribution are unknown, and parametric models.

One of the oldest life duration models are life tables which constitute a group of nonparametric models. Life tables are a theoretical model of population, which size gradually decreases as a consequence of dying. Therefore having the radix population l_0 (in life tables l_0 is set at 100 000) and given the number of persons dying during a year at the age of $x - d_x$, it is possible to calculate the number of survivors from to age $x - l_x$. The basic parameters of life tables are probabilities of duration of a person at the age of x and the probabilities of dying during a year of person aged x (therefore

[1] In order to preserve traditional notations the concept of „age" of x years is used instead of calendar time t, *hence the force of mortality is* μ_x.

functions $S(t)$ and $F(t)$, which in a traditional notation take forms p_x i q_x)[2]. Another, equally often quoted parameter of life tables is life expectancy at age x – e_x.

Among the parametric models of force of mortality, we can gather into a group of models of desirable function graph those models proposed in [9]:

– logistic model: $$m_x^{(L)} = c + \frac{a \cdot e^{b \cdot x}}{1 + d \cdot (e^{b \cdot x} - 1)}, \tag{4}$$

– Kannisto model: $$m_x^{(K)} = \frac{a \cdot e^{b \cdot x}}{1 + a \cdot (e^{b \cdot x} - 1)}, \tag{5}$$

– Coale-Kisker model (exponential-parabolic)[3]:

$$m_x^{(WP)} = e^{a + b \cdot x + c \cdot x^2}, \tag{6}$$

and models proposed in [5]:

– hyperbolic tangent model:

$$m_x^{(T)} = A \cdot tgh(B \cdot x + C) + D, \tag{7}$$

between formulae (4) and (7) the following dependencies appear (compare [5]):

$$a = \frac{2 \cdot A}{1 + e^{-2 \cdot C}}; \quad b = 2 \cdot B; \quad c = D - A; \quad d = \frac{1}{1 + e^{-2 \cdot C}},$$

where small letters refer to logistic model, capital letters refer to hyperbolic tangent model,

– simplified logistic model:

$$m_x^{(LU)} = \frac{a \cdot e^{b \cdot x}}{1 + d \cdot (e^{b \cdot x} - 1)}. \tag{8}$$

2. Analysis of mortality among people aged 80 and more in the EU countries of the Baltic Sea Region

The analysis of mortality among the old old (people aged 80 and more) have been based on data gathered in the database: *Kannisto-Thatcher Database on Old Age Mortality at the Max Planck Institute for Demographic Research*, concerning mortality of people aged 80 and more.

At first, selected parameters of life tables for eight EU countries of the Baltic Sea Region: Denmark, Estonia, Finland, Germany, Latvia, Lithuania, Poland and Sweden for period 1990-2004 have been estimated. Figures 1-2 show probabilities of dying q_x for females and males at the age of over 80, whereas figures 3-4 display life expectancy e_x for females and males aged 80 and more in the analyzed countries.

Studying the values of life duration parameters it can be noticed that both, probabilities of dying (also force of mortality) and life expectancy for all analyzed countries are of a slight variation in younger age groups among the old old. For people aged 95-100 and more the analyzed parameters reveal at times very strong variation. Main reason for this variation are small-sized age intervals, for both persons alive and dead.

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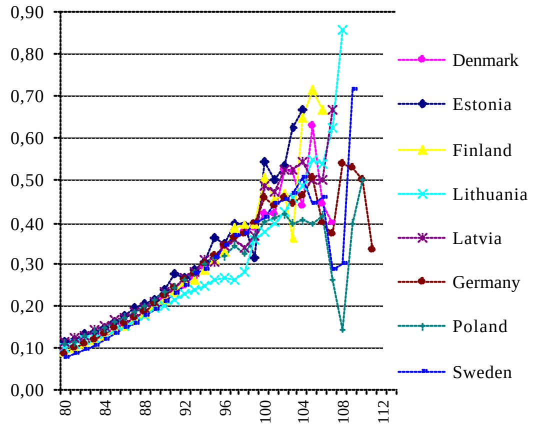
[2] In order to preserve traditional notations, instead of calendar time (t) a concept of „age" (x) will be used.

[3] Model proposed by Coale and Kisker is based on an assumption that rate of changes of deaths coefficient equals the difference of logarithms of deaths coefficients for elderly people, which in consequence leads to function of quadratic form [10].



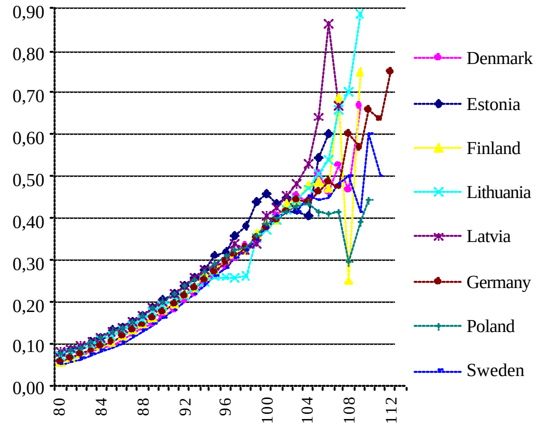
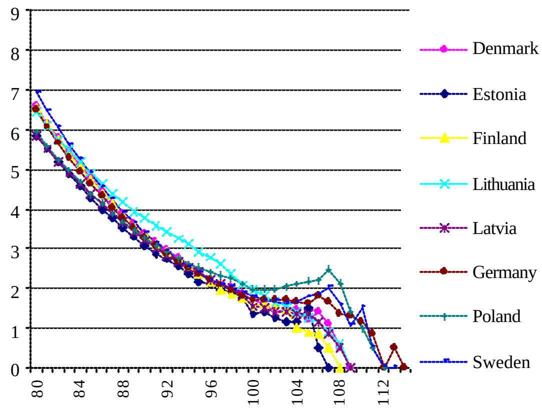


Fig. 1. Probability of dying q_x for men aged 80 and more in the countries of the Baltic Sea Region (1990-2004)

Source: own research.

Fig. 2. Probability of dying q_x for women aged 80 and more in the countries of the Baltic Sea Region (1990-2004)

Source: own research.



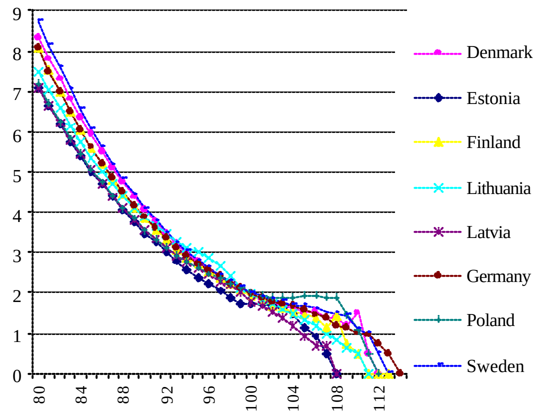


Fig. 3. Life expectancy e_x for men aged 80 and more in the countries of the Baltic Sea Region (1990-2004)

Source: own research.

Fig. 4. Life expectancy e_x for women aged 80 and more in the countries of the Baltic Sea Region (1990-2004)

Source: own research.

For better description of the differences in mortality of the old old in the EU countries of the Baltic Sea Region table 1 presents the values of life expectancy in five-years age groups for people aged 80 and more. Calculation of life expectancy in five-years age groups was based on estimated probabilities of dying in five-years age intervals.

To summarize, parameters of life tables – full and abridged (for five-years age groups) unequivocally divide the analyzed countries into two groups. First group consists of countries of so called "old" EU: Denmark, Finland, Germany and Sweden, for which parameters of life expectancy for people aged 80 and more are higher by over half a year in case of men and by over one year in case of women. Among the EU countries of the Baltic Sea Region the highest values of parameters of life expectancy have been observed in 1990-2004 in Sweden, while the lowest in Latvia and Estonia. The difference in life expectancy for people aged 80 and more in the country of the highest and the country of the lowest values were – for males over a year, for females over 1,5 year.

Table 1. Life expectancy in five-years age groups in EU countries of the Baltic Sea Region in 1990-2004

Age group	Denmark	Estonia	Finland	Lithuania	Latvia	Germany	Poland	Sweden
	Males							
80-84	6,90	6,07	7,10	6,37	5,82	6,76	6,29	7,40
85-89	5,22	4,67	5,38	5,05	4,57	4,92	4,88	5,47
90-94	4,03	3,90	4,25	4,29	3,88	4,12	4,21	4,18
95-99	3,20	3,21	3,43	3,62	3,20	3,44	3,62	3,35
100-104	2,73	2,50	2,75	2,63	2,50	2,98	3,16	2,99
105-109	2,50		2,50	2,50		2,82	3,63	2,61
110-114						2,50	2,50	2,50
	Females							
80-84	8,60	7,31	8,56	7,72	7,14	8,30	7,50	9,12
85-89	6,33	5,33	6,23	5,62	5,22	5,95	5,52	6,59
90-94	4,70	4,14	4,70	4,40	4,15	4,72	4,47	4,84
95-99	3,70	3,23	3,80	3,52	3,41	3,84	3,75	3,79
100-104	3,15	2,61	3,22	2,58	2,79	3,31	3,31	3,22
105-109	2,86	2,50	2,87	2,50	2,50	2,96	3,45	3,03
110-114	2,50		2,50			2,50	2,50	2,50

Source: own research.

3. Mortality models of people aged 80 and more in the EU countries of the Baltic Sea Region

Since the analysis of parameters of life tables has shown strong variation of measures, for people at the age of over 90 more advanced analysis was carried out, which was based on selected parametric life duration models.

Out of quoted models (4) – (8) Coale-Kisker model (6) has been chosen for the description of mortality among old old people in the EU countries. This Coale-Kisker model is based on an assumption that rate of changes of deaths coefficient equals the difference of logarithms of deaths coefficients for elderly people, which in consequence leads to function of quadratic form:

$$\ln(m_x) = a + bx + cx^2 \text{, dla } c < 0. \qquad (9)$$

The main advantage of this model is its simplicity and the fact that it can be applied using both cross-section data as well as cohort data. This model was positively verified on data coming from Japan and Sweden for people aged 85 and more [10].

Table 2 presents estimated values of structural parameters Coale-Kisker model. It turned out that not for every country force of mortality models are statistically significant. In the case of men the modelling made sense for Denmark, Germany, Poland and Sweden, whereas in the case of women – for each of the analysed countries except for Latvia and Lithuania.

Table 2. Coale-Kisker model estimation results (9)*)

Males				Females			
a	*b*	*c*	R^2	*a*	*b*	*c*	R^2
Denmark							
-21,933 (-5,806)	0,367 (-4,521)	-0,0015 (-3,557)	0,9625	-21,199 (-11,035)	0,328 (-8,029)	-0,0012 (-5,744)	0,9916
Estonia							
-2,826 (-3,673)	-0,06 (-0,645)	0,0008 (-1,584)	0,9705	-20,274 (-7,169)	0,321 (-5,254)	-0,0012 (-3,798)	0,9846
Finland							
-8,155 (-1,842)	0,058 (-0,610)	0,0002 (-0,349)	0,9649	-24,931 (-4,367)	0,411 (-3,386)	-0,0017 (-2,639)	0,9271
Lithuania							
5,268 (-1,541)	-0,222 (-3,032)	0,0016 (-4,160)	0,9693	-2,076 (-0,568)	-0,079 (-1,012)	0,0009 (-2,240)	0,971
Latvia							
-8,984 (-3,412)	0,093 (-1,636)	-0,0001 (-0,285)	0,9801	-3,292 (-0,752)	-0,054 (-0,571)	0,0008 (-1,606)	0,9666
Germany							
-29,31 (-10,606)	0,531 (-9,116)	-0,0024 (-8,025)	0,9609	-18,841 (-12,172)	0,28 (-8,620)	-0,001 (-5,862)	0,9924
Poland							
-28,674 (-5,091)	0,536 (-4,496)	-0,0026 (-4,111)	0,7859	-31,164 (-14,162)	0,568 (-12,175)	-0,0026 (-10,763)	0,9783
Sweden							
-29,654 (-5,273)	0,532 (-4,742)	-0,0024 (-4,089)	0,9107	-29,654 (-18,490)	0,508 (-15,014)	-0,0022 (-12,389)	0,9926

*) Values of Student's t-statistics have been put in brackets underneath the parameters estimates. The shadowed values of parameters estimates indicate lack of statistical significance of a given parameter.
Source: own research.

Since existing of inflexion point is typical feature of functions used in modelling mortality of the old old, its value was calculated for significant Coale-Kisker models (compare fig. 5 and 6). What is more, mortality reaches the highest values in the purlieus of inflexion point – sometimes the value of this point is called demographic threshold of old [1].

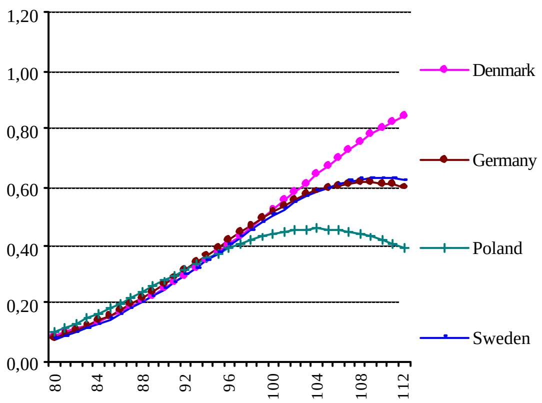


Fig. 5. Force of mortality μ_x for men aged 80 and more in the countries of the Baltic Sea Region (1990-2004)
Source: own research.

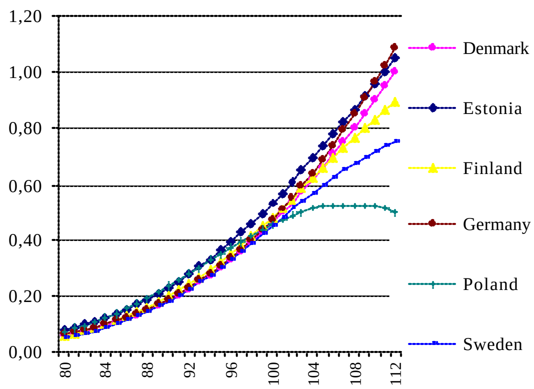


Fig. 6. Force of mortality μ_x for women aged 80 and more in the countries of the Baltic Sea Region (1990-2004)
Source: own research.

The threshold values for men in 1999-2004 reached the following values: Denmark: 100 years, Germany: 94 years, Poland: 90 years and Sweden: 95 years. For women – Denmark: 112 years, Estonia: 108 years, Finland: 104 years, Germany: 120 years, Poland: 93 years and Sweden: 100 years.

Summary

Mortality among people aged 80 and more in EU countries of the Baltic Sea Region reveals some differentiation. The values of life tables parameters can be clearly divided into two groups of countries of so called „old" and „new" EU. For countries with the lowest values of life tables parameters the trial of modelling mortality with Coale-Kisker model was unsuccessful.

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