

# Gradual Foreign Market Commitment under Uncertainty: The Case of International Joint Ventures and Subsequent Buyouts

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**Abstract** Real option analysis has been applied to strategies of market entry and global expansion, predominantly in combination with the formation of alliances and joint ventures. Albeit joint venturing based on option pricing theory is studied in both disciplines, financial economics and strategic management, the link between these strains of literature is not well developed. We use a compound option framework to analyze the impact of decision contingency and learning on formation and duration of joint ventures. In particular, critical thresholds are presented which allow to characterize the conditions under which termination takes place and to estimate the duration of international joint ventures revealing a novel perspective on existing empirical findings. While the model also provides a number of new testable predictions the presented closed-form solution can help managers to structure explicit option clauses in JV contracts more efficiently.

**Keywords** Foreign direct investment, multinational enterprise, sequential investments, entry mode, international joint venture, real options.

**JEL classification numbers:** D43, F23, L13, P31.

**Acknowledgements** Much of the research for this paper was undertaken while the author was a visiting scholar at the Kenan-Flagler Business School, University of North Carolina at Chapel Hill. I am grateful to B. Michael Gilroy, Jeffrey J. Reuer, Stephen Tallman, and Udo Broll for helpful comments and discussions. The thoughtful reviews by two anonymous referees are greatly acknowledged. Any remaining errors are the sole responsibility of the author.

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## 1 Introduction

Market globalization has transformed the nature of corporate operations. Scholars have recently called this period an era of alliance capitalism indicating the importance for firms to lever the assets, skills, and experiences of globally dispersed partners (see e.g. Dunning 1997). Besides their widespread use alliances and joint ventures (JVs) in particular, however, show great heterogeneity regarding their instability rates. While some JVs last considerably long, e.g. DowCorning is more than 50, FujiXerox more than 30 years old, others are terminated shortly after their foundation. Yet the management literature has acknowledged that equating instability with failure may be inaccurate. For example, more than 80% of the international alliances studied by Bleeke and Ernst (1991) ended in acquisitions and not in abandonment. However, models that address the decision to enter a JV have been too static and thus fail to take proper account of the strategic intent, i.e. to expand subsequently in the host country. Moreover, important key parameters, e.g. the uncertainty that is created by the volatility in the international business environment or the irreversibility issues of most foreign resource commitments, have been neglected. In particular, there is a lack of in-depth research in the international business and management literature concerning the following questions. First, what triggers the switching of modes and under which circumstances does the firm expand an international joint venture (IJV) from a dynamic viewpoint? Second, while there is still a debate ongoing with respect to the choice of optimal degree of foreign ownership, current research fails to provide clear answers about how this choice is affected by uncertainty and its future resolution due to learning and knowledge accumulation.

## 2 Literature Review

A joint venture (JV) is an agreement between two or more legally independent entities which pool their capabilities and resources to form a shared business. The JV becomes an international joint venture (IJV) if at least one foreign partner is involved. While factors affecting JV formation have received abundant attention, the processes of JV evolution have received relatively scant attention.<sup>1</sup> By JV evolution we mean a JV's development along its life cycle, i.e. formation, operation, and termination (Child et al. 2005). Given this context, the bulk of literature has investigated JV evolution by means of empirical methods. In particular, those studies have mainly taken on an outcome oriented perspective and used instability, i.e. termination as a suitable criteria.<sup>2</sup> In general, the primary question is, how long do JVs survive beyond their formal announcement and which factors affect their instability. Some common factors exist that appear to be conducive to the transitional phenomenon of JVs and IJVs in particular.<sup>3</sup> These are e.g. equity structure, uncertainty in the external economic environment, cultural distance, experience and learning capabilities among others. Great heterogeneity, however, exists with respect to whether these factors influence stability in a positive or negative way.<sup>4</sup> Exemplary, the findings remain to a great extent ambiguous whether parity or majority/minority equity partnerships are more stable (see e.g., Blodgett 1992; Dhanaraj and Beamish 2004). Like-

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<sup>1</sup> For a comprehensive primer on the research on strategic alliances see e.g. Contractor and Lorange (1988) or Todeva and Knoke (2005).

<sup>2</sup> Instability may also arise due to reasons other than termination. Some scholars take on a process oriented perspective. Here, changes in the ownership structure, e.g. due to contractual renegotiation are viewed as sign of instability. For a detailed discussion see e.g. Reuer and Miller (1997), Yan and Zeng (1999), or Larimo (2007).

<sup>3</sup> For a synopsis, see e.g. Yan and Zeng (1999).

<sup>4</sup> See, e.g. Beamish (1988), Park and Russo (1996), Meschi (2005), and Mata and Portugal (2007).

wise, there is a lack of clarity whether external uncertainty has an enhancing effect on the survival of JVs (see, e.g., Kogut 1991; Hennart et al. 1998; Luo and Park 2004).

Viewing JVs in terms of their ability to generate subsequent managerial choices brought about that looking at JVs from a real option perspective has surged in recent years.<sup>5</sup> Real options are generated when existing assets, resources or capabilities allow preferential access to subsequent investment opportunities, may they be immediately born out of the initial commitment or generated in the future (Bowman and Hurry 1993).<sup>6</sup> The literature has revealed that those real investment options are economically valuable when investments are made under condition of considerable uncertainty and when they are (partial) irreversible, i.e. their initial pecuniary value cannot be fully recovered once in use. Given such a setting, it is well recognized that a real option perspective greatly advances the understanding of the economic logic behind the behavioral process of incremental resource commitments and market entry respectively.<sup>7</sup>

Kogut (1991) was among the first to apply this concept to the theory of foreign direct

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<sup>5</sup> This has also facilitated a paradigm shift in the domain of JV research namely that the objective in governance choice is not motivated by minimizing transaction cost but maintaining flexibility.

<sup>6</sup> Put it differently, real option theory suggests viewing real investments as options that buy the firm the right to make investments later, the right to defer or alter the scale or to initiate subsequent investments. A detailed introduction to real options is given by Trigeorgis (1998) and Dixit and Pindyck (1994). For models tailored to particular characteristics of investment projects see e.g. Brennan and Schwartz (1985), McDonald and Siegel (1986) or Cortazar et al. (1998), among others.

<sup>7</sup> See e.g. Bowman and Hurry (1993), Buckley and Casson (1998), and Childs et al. (1998). For a general discussion of the decision complexities and contingencies manager's have to face in a cooperative venture see e.g. Tallman and Shenkar (1994).

investment and JV initiation, in particular.<sup>8</sup> Hence, a JV can be understood as a call option that limits the downside risk of the firm while allowing managers to benefit from positive developments in the future once they materialize. Consequently, the termination of an IJV does not indicate its failure but the exploitation of its flexibility; results that have found great empirical support lately. For instance, Kogut (1991) and Folta and Miller (2002) find empirical evidence that uncertainty is an important driver in timing the partner buyout and terminating the JV in order to capitalize on the growth option.<sup>9</sup> In particular, buying out the partner is more likely the lower the initial equity stake which challenges classic findings that have so far predicted that most majority-owned JVs became WOS later on (see, e.g. Gomes-Casseres 1987). Moreover, the diffusion of the real option logic has advanced JV contract design by originating explicit buyout/divestment clauses (Chi and Seth 2002). Surprisingly, however, the number of reported incidents of explicit option clauses in studies concerning JVs is almost negligible. Option-to-acquire clauses accounted only for 1% of the sample size and this fraction was almost constant over time (Reuer and Tong 2005). This can be an indicator of the difficulties of management to determine a fair option premium especially because there are no closed form solutions like the Black-Scholes formula at present to value such complex investments.<sup>10</sup>

While these empirical studies are more concerned with the implications for outcome and performance respectively few have approached the real option features of JVs in terms of rigorous theoretical modeling. Pennings and Sleuwaegen (2005) design an option model where both the timing of market entry and the entry mode are determined simultaneously. The main focus lies on the timing decision whether to form a JV or a wholly-owned subsidiary (WOS) which is impacted by transfer prices, amount of equity share, market

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<sup>8</sup> See also Kogut and Kulatilaka (1994).

<sup>9</sup> See also McGrath (1997), Reuer and Leiblein (2000), Dyer et al. (2004), and Tong et al. (2008).

<sup>10</sup> See, e.g. Graham and Harvey (2001). For an overview on the obstacles to real option valuation in a management context see, e.g. Lander and Pinches (1998) and Miller and Shapira (2004).

structure, and the degree of governmental regulation. The transitional nature of JVs is, however, not addressed. Chi (2000) extends the setting of JV specific real options rights by implementing explicitly two termination options. Hence, the presented model is a first attempt to capture the transitional nature of JVs.<sup>11</sup> In addition, he assumes that continued collaborating lowers the degree of uncertainty in the partner's JV valuation. However, in his framework the impact of learning is only discussed in the context of divergence of economic value. Neither is the impact on the choice of JV formation nor on the expected duration of the JV discussed. Habib and Mella-Barral (2007) link the acquisition of knowledge to the dissolution of JVs and estimate their duration by means of real option analysis. In particular, acquired knowhow increases the profitability of a partner's separate operation of the joint asset. Consequently, if separate operation is more profitable than joint operation the partner that possesses the superior capabilities will exercise the option to dissolve the collaboration. The findings reveal that the duration is positively affected by the uncertainty about the learning conditions while it decreases with the ease by which the partner can acquire the knowledge.

Despite its potential to contribute to the analysis of the evolutionary sequence patterns of JVs the real option induced literature, however, so far neglects the impact of interdependencies and the compoundness of subsequent options an issue that deserves particular attention especially in the context of a learning process. Over and above the attempts of the current literature, the goal of this paper is twofold: To model a market entry under uncertainty in a continuous time setting given the observed fact of an evolutionary expansion sequence via an IJV, and to show the impact of uncertainty resolution on the dynamics of IJVs. The findings contrast with classical real option induced market entry results where high uncertainty always facilitates waiting. Further, high uncertainty exerts a positive impact on duration of the IJV supporting the hypothesis that IJVs are a means

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<sup>11</sup> See also Chi and McGuire (1996).

to circumvent economic uncertainties (see e.g. Luo and Park 2004; Meschi 2005). We show that the question whether parity or unequal ownership structures are more stable cannot be answered in isolation. Rather, the property of asset specificity causes an interrelation of equity share and uncertainty which calls for a conjoint analysis. The results furthermore contradict the findings of Chi and McGuire (1996) who note that the learning potential combined with the options embedded in a joint venture enhance the economic value of JVs. In particular, when learning is possible the growth option value is alleviated and the propensity to increase in collaborative venturing decreases which supplements recent empirical findings of Cuypers and Martin (2006). Finally, our findings support the literature that states that learning capabilities increase the instability of IJVs. The remainder of the paper is structured as follows. In section three, we will present the model: a three-phase market entry sequence subject to exogenous and endogenous uncertainty. The main results are presented in section four, while section five summarizes the main findings and provides a synopsis of major comparative-static results. Finally, section six concludes and provides stimuli for future research.

### 3 The Model

Consider a firm that has to decide whether or not to enter a new geographical market via a JV. The investment is subject to two sources of risk. First, the firm has to consider the uncertainty about the future development of the JV's value  $\tilde{V}$ . Second, since the investment is made abroad its value is also subject to the development of the corresponding exchange rate  $E(t)$ . For simplicity, we will model the time-varying dynamics of both  $\tilde{V}(t)$  and  $E(t)$  by means of two geometric Brownian motions (gBMs). Assuming a perfect capital market, the existence of a unique martingale measure  $Q$  is guaranteed and the value of the investment expressed in domestic currency  $V(t) \equiv \tilde{V}(t)E(t)$  can be expressed

by the following stochastic differential equation:<sup>12</sup>

$$dV(t) = (r - \delta)V(t)dt + \Sigma_0 V(t)dB^Q(t), \quad (1)$$

where  $r \in \mathbb{R}^+$  is the risk-free interest rate,  $\delta \in \mathbb{R}^+$  represents the opportunity cost of waiting, and  $dB^Q$  indicates a Wiener process with non-zero drift given a martingale measure  $Q$ . In addition,  $\Sigma_0 \in \mathbb{R}^+$  designates the variance of  $dV/V$  and equals  $\sqrt{\sigma_0^2 + \sigma_E^2 + 2\sigma_0\sigma_E\rho}$  with  $\sigma_0$  and  $\sigma_E$  as the corresponding measures for project and exchange rate uncertainty and instantaneous correlation  $\rho$ . We will use the domestic value  $V(t)$  for any further consideration of the value of the claims.

It is assumed that the market entry follows a three stage process, and that each stage is connected to some sort of sunk costs.<sup>13</sup> The first phase to be considered represents the initiation phase of an IJV, e.g. the decision whether or not to establish a physical presence by holding either a minority, majority or equal stake in the IJV. Let  $\epsilon \in \mathbb{R}^+$  refer to the initial equity stake the firm wishes to invest in which is given exogenously.<sup>14</sup> Furthermore, let  $\bar{\epsilon}$  be a host-country unique upper boundary with respect to the overall equity a foreign firm is allowed to hold.<sup>15</sup> Then the value of such a market entry for the

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<sup>12</sup> For a detailed derivation see the Appendix.

<sup>13</sup> Moreover, we will assume that throughout the duration of each stage the option rights are exclusive and that there are no problems of forfeiture or expiration limits with regard to exercising the investment option.

<sup>14</sup> To ensure that the venture will be an equity JV we set the minimum equity stake at 5% (see, e.g. Gomes-Casseres 1987).

<sup>15</sup> The restriction may, for example, be put in place to prevent foreign firms from taking complete control over national champions.



firm is equal to:<sup>16</sup>

$$W(V) = E^Q \left[ \frac{[\epsilon V + F((\bar{\epsilon} - \epsilon)V) - I_1]^+}{e^{r(t_1 - t_0)}} | \mathcal{F}_0 \right], \quad (2)$$

given the filtration  $\mathcal{F}_0$ . Here,  $I_1$  designates the costs assigned to the market entry and  $F((\bar{\epsilon} - \epsilon)V)$  represents the value of flexibility due to subsequent routes of action.

Due to the fact that an IJV involves co-ownership as well as co-management, both partners are exposed to the risk that obstacles will arise over the course of the project which may hamper a smooth decision-making process. Consequently, it is worthwhile considering a certain period of time for the partners to become acquainted and explore whether they can work together for the sake of the venture (Gilroy 1993). We will designate this time with  $T = t_2 - t_1$ . It is reasonable to assume that during this period the value is conceivably susceptible to two types of uncertainty. The first type of uncertainty is exogenous to the firm, i.e. it is largely independent of what the firm does. The second type, however, is endogenous and can be reduced by the actions of the firm. More precisely, only a commitment to the JV reveals information about the true extent of the partners' capabilities, the resulting synergies, and about the long-term objectives of the partner.<sup>17</sup> Similar, the firm can dismantle information asymmetry about the potential acquisition target. We neglect uncertainty about this component and will assume that if present, the time-varying trend is deterministic and negatively correlated with the collaboration period.<sup>18</sup> Consequently, the longer firms collaborate the more this kind of endogenous

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<sup>16</sup> This view is justified by the fact that if the gains are always divided according to some fixed proportion then the situation is identical with one firm taking the active role (Buckley and Casson 1996, p. 873).

<sup>17</sup> See Chi and Seth (2002, p. 75f.). For further details on the two types of uncertainty see also Roberts and Weitzman (1981) and Folta (1998).

<sup>18</sup> Research on financial option pricing has revealed that volatility is not constant over the life of an option. Hence, advanced models have treated volatility as a stochastic variable. See, e.g.

uncertainty is reduced due to knowledge accumulation and organizational learning at multiple levels, however, only up to a certain level indicating that uncertainty cannot be resolved completely. Hence, the dynamics of  $V(t)$  in  $t \in [t_1, t_2]$  will change to:

$$dV(t) = (r - \delta)V(t)dt + (\phi\Sigma(t) + (1 - \phi)\Sigma_0)V(t)dB^Q(t), \quad (3)$$

with  $\Sigma(t) \in \mathbb{R}^+$  as a time-varying function and  $\phi$  being either zero if no uncertainty resolution is possible, and one if uncertainty reduction is possible. For simplicity, we will assume that learning only affects the future development of the JV's value  $V(t)$  so that  $\Sigma^2(t)$  results to:

$$\Sigma^2(t) = \sigma^2(t) + \sigma_E^2 + 2\sigma(t)\sigma_E\rho. \quad (4)$$

Here,  $\sigma(t)$  is expressed as:<sup>19</sup>

$$\propto \int_{t_1}^{t_2} \sigma_0 e^{-\gamma\tau} d\tau, \quad (5)$$

with  $\gamma \in \mathbb{R}^+$  indicating the different abilities of firms to absorb information and/or knowledge spill-overs over time. Subsequent to this second period, the firm can decide to exercise the option and expand its foreign market presence by acquiring the rest of the equity stake, i.e.  $(\bar{\epsilon} - \epsilon)V$  in the third phase. For simplicity we will assume that the dynamics of  $V(t)$  are again described by equation (1), however, with the possibility to take account of uncertainty reduction. Thus, in the case of uncertainty reduction ( $\phi=1$ ) the volatility is given by  $\sigma(t_2) \equiv \sigma_1$  and  $\Sigma(\sigma(t_2)) \equiv \Sigma_1$ .<sup>20</sup> If no uncertainty reduction Hull (2009).

<sup>19</sup> Following Spence (1983), we will assume that learning and knowledge accumulation effects increase exponentially over time while collaborating. Following suggestions of several authors we will assume that (passive) learning deterministically decreases the instantaneous variance. See, e.g. Majd and Pindyck (1989), Childs and Triantis (1999), Chi (2000), and Martzoukos (2000).

<sup>20</sup> Since  $\sigma(t_2)$  is fixed, uncertainty does not diminish completely the longer firms collaborate.

is possible ( $\phi=0$ ) then  $\sigma$  remains at the initial level, i.e.  $\sigma_0$  with corresponding overall uncertainty  $\Sigma_0$ . Hence, we can express this formally as:

$$dV(t) = (r - \delta)V(t)dt + (\phi\Sigma_1 + (1 - \phi)\Sigma_0)V(t)dB^Q(t), \quad (6)$$

Formalizing the optimization problem in this manner is similar to the analytics of a compound option. A compound option simply refers to option rights on options and has been analyzed by Geske (1979). It may be demonstrated that for each stage there is a threshold value at which it is optimal for a firm to exercise the investment option.<sup>21</sup> The following section briefly summarizes the trigger values that illustrate when it is optimal for a firm to trigger the first, second, and third stage of the market entry via an IJV.

## 4 Results

In the following the main findings resulting from the previous introduced assumptions are summarized. It is worthwhile stating that the solution of the problem is generally determined recursively. However, it is convenient to present the results in a forward looking fashion.

**Proposition 1** *The flexibility for a firm to enter the market via an IJV is determined by:*

$$\begin{aligned} W(V) = & \epsilon V_0 e^{-\delta t_1} N(d_6) - I_1 e^{-rt_1} N(d_6 - \Sigma_0 \sqrt{t_1}) \\ & + (\bar{\epsilon} - \epsilon) V_0 e^{-\delta t_2} \left( (1 - \phi) M(h_1 + \Sigma_0 \sqrt{t_1}, k_1; \rho) + \phi M(h_1 + \Sigma_0 \sqrt{t_1}, k_1; \hat{\rho}) \right) \\ & - I_2 e^{-rt_2} \left( (1 - \phi) M(h_1, k_2; \rho) + \phi M(h_1, k_2; \hat{\rho}) \right) \\ & + (1 - \phi) A V_0^{\beta_1} (M(h_2, k_3; -\rho) - M(h_2, k_5; -\rho)) \\ & + \phi B V_0^{\gamma_1} e^{\Omega_1 T} e^{-rt_2} e^{\Omega_2 t_1} (M(h_2, k_3; -\hat{\rho}) - M(h_2, k_5; -\hat{\rho})) \\ & - \bar{I} e^{-rt_2} \left( (1 - \phi) M(h_1, k_4; \rho) + \phi M(h_1, k_4; \hat{\rho}) \right), \end{aligned} \quad (7)$$

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<sup>21</sup> The derivation of the threshold values is given in the appendix.

with  $\hat{\rho} = \Gamma\sqrt{t_1/t_2}$ ,  $\rho = \sqrt{t_1/t_2}$ ,  $\Omega_2 = \frac{1}{2}\gamma_1\Sigma_0^2(\gamma_1 - 1) + (r - \delta)\gamma_1$ ,  $\hat{\Sigma}^2 = \frac{1}{2}\frac{(\sigma_1^2 - \sigma_0^2)}{\ln(\sigma_1/\sigma_0)} + \sigma_E^2 + \frac{2\sigma_E(\sigma_1 - \sigma_0)\rho}{\ln(\sigma_1/\sigma_0)}$ , and  $\Gamma = \frac{\Sigma_0}{\hat{\Sigma}\sqrt{1 + ((\Sigma_0/\hat{\Sigma})^2 - 1)(t_1/t_2)}}$ .  $V_0$  states the value of the IJV at time  $t = 0$ .

Here,  $M(\dots)$  designates the bivariate cumulative standard normal distribution with:

$$\begin{aligned} d_6 &= (1 - \phi) \left( \frac{\ln\left(\frac{V_0}{V_1^*}\right) + (r - \delta + \frac{1}{2}\Sigma_0^2)t_1}{\Sigma_0\sqrt{t_1}} \right) + \phi \left( \frac{\ln\left(\frac{V_0}{V_{1L}^*}\right) + (r - \delta + \frac{1}{2}\Sigma_0^2)t_1}{\Sigma_0\sqrt{t_1}} \right), \\ k_1 &= (1 - \phi) \left( \frac{\ln\left(\frac{V_0}{V^\infty}\right) + (r - \delta + \frac{1}{2}\Sigma_0^2)t_2}{\Sigma_0\sqrt{t_2}} \right) + \phi \left( \Gamma \left( \frac{\ln\left(\frac{V_0}{V^\infty}\right) + (r - \delta + \frac{1}{2}\hat{\Sigma}^2)t_2 + \frac{1}{2}(\Sigma_0^2 - \hat{\Sigma}^2)t_1}{\Sigma_0\sqrt{t_2}} \right) \right), \\ k_2 &= (1 - \phi) \left( \frac{\ln\left(\frac{V_0}{V^\infty}\right) + (r - \delta - \frac{1}{2}\Sigma_0^2)t_2}{\Sigma_0\sqrt{t_2}} \right) + \phi \left( \Gamma \left( \frac{\ln\left(\frac{V_0}{V^\infty}\right) + (r - \delta - \frac{1}{2}\hat{\Sigma}^2)t_2 - \frac{1}{2}(\Sigma_0^2 - \hat{\Sigma}^2)t_1}{\Sigma_0\sqrt{t_2}} \right) \right), \\ k_3 &= (1 - \phi) \left( \frac{\beta_1 \ln\left(\frac{V^\infty}{V_0}\right) - (r + \frac{1}{2}\Sigma_0^2\beta_1^2)t_2}{\Sigma_0\beta_1\sqrt{t_2}} \right) + \phi \left( \Gamma \left( \frac{\beta_1 \ln\left(\frac{V^\infty}{V_0}\right) - (\Omega_2 + \frac{1}{2}\Sigma_0^2\gamma_1^2)t_1 - (\Omega_1 + \frac{1}{2}\hat{\Sigma}^2\gamma_1^2)T}{\Sigma_0\gamma_1\sqrt{t_2}} \right) \right), \\ k_4 &= (1 - \phi) \left( \frac{\ln\left(\frac{V_0}{V}\right) + (r - \delta - \frac{1}{2}\Sigma_0^2)t_2}{\Sigma_0\sqrt{t_2}} \right) + \phi \left( \Gamma \left( \frac{\ln\left(\frac{V_0}{V}\right) + (r - \delta - \frac{1}{2}\hat{\Sigma}^2)t_2 - \frac{1}{2}(\Sigma_0^2 - \hat{\Sigma}^2)t_1}{\Sigma_0\sqrt{t_2}} \right) \right), \\ k_5 &= (1 - \phi) \left( \frac{\beta_1 \ln\left(\frac{V}{V_0}\right) - (r + \frac{1}{2}\Sigma_0^2\beta_1^2)t_2}{\Sigma_0\beta_1\sqrt{t_2}} \right) + \phi \left( \Gamma \left( \frac{\beta_1 \ln\left(\frac{V}{V_0}\right) - (\Omega_2 + \frac{1}{2}\Sigma_0^2\gamma_1^2)t_1 - (\Omega_1 + \frac{1}{2}\hat{\Sigma}^2\gamma_1^2)T}{\Sigma_0\gamma_1\sqrt{t_2}} \right) \right), \\ h_1 &= (1 - \phi) \left( \frac{\ln\left(\frac{V_0}{V_1^*}\right) + (r - \delta - \frac{1}{2}\Sigma_0^2)t_1}{\Sigma_0\sqrt{t_1}} \right) + \phi \left( \frac{\ln\left(\frac{V_0}{V_{1L}^*}\right) + (r - \delta - \frac{1}{2}\Sigma_0^2)t_1}{\Sigma_0\sqrt{t_1}} \right), \\ h_2 &= (1 - \phi) \left( \frac{\beta_1 \ln\left(\frac{V_0}{V_1^*}\right) + (r + \frac{1}{2}\Sigma_0^2\beta_1^2)t_1}{\Sigma_0\beta_1\sqrt{t_1}} \right) + \phi \left( \frac{\gamma_1 \ln\left(\frac{V_0}{V_{1L}^*}\right) + (\Omega_2 + \frac{1}{2}\Sigma_0^2\gamma_1^2)t_1}{\Sigma_0\gamma_1\sqrt{t_1}} \right). \end{aligned}$$

**Proof 1** See Appendix.

Proposition 1 states the option value to form an IJV in  $t_1$  years from now. This flexibility value is comprised of two parts. While the first part, i.e. the first two terms of the solution correspond to the Black-Scholes formula and emphasize the value of waiting to invest the second part, i.e. the remaining terms value the subsequent flexibility. In particular, this latter part captures not only the value of the option to buyout the partner at some point in the future but also assesses the impact of knowledge accumulation on this kind of flexibility. As the result indicates, besides uncertainty, costs, amount of equity share, and time the option value is sensitive to the subsequent threshold value  $V_1^*$  which indicates

optimal market entry. Hence, the following proposition summarizes when it is optimal for the firm to form an IJV.

**Proposition 2** *The firm will enter into an IJV, i.e. exercise the first stage, if the IJV's asset value  $V(t)$  at  $t_1$  reaches at least an optimal trigger value  $V_1^*$  determined by:*

$$\epsilon V_1^* + F(V_1^*) - I_1 \stackrel{!}{=} 0, \quad (8)$$

with  $I_1$  as the initial investment cost. The value of the subsequent option  $F(\dots)$ , i.e. the flexibility for further expansion, is given by:

$$\begin{aligned} F = & (\bar{\epsilon} - \epsilon)V_1 e^{-\delta T} N(d_1) - I_2 e^{-rT} N(d_2) + (1 - \phi) A V_1^{\beta_1} (N(d_3) - N(d_4)) \\ & + \phi e^{(\Omega_1 - r)T} B V_1^{\gamma_1} (N(d_3) - N(d_4)) - \bar{I} e^{-rT} N(d_5), \end{aligned} \quad (9)$$

with  $V_1$  as the value of the project at time  $t_1$ , and  $\Omega_1 = \frac{1}{2}\gamma_1 \hat{\Sigma}^2(\gamma_1 - 1) + (r - \delta)\gamma_1$ .

Here,  $N(\dots)$  designates the cumulative normal distribution and

$$\begin{aligned} d_1 &= (1 - \phi) \left( \frac{\ln\left(\frac{V_1}{V^\infty}\right) + (r - \delta + \frac{1}{2}\Sigma_0^2)T}{\Sigma_0\sqrt{T}} \right) + \phi \left( \frac{\ln\left(\frac{V_1}{V^\infty}\right) + (r - \delta + \frac{1}{2}\hat{\Sigma}^2)T}{\hat{\Sigma}\sqrt{T}} \right), \\ d_2 &= (1 - \phi) \left( d_1 - \Sigma_0\sqrt{T} \right) + \phi \left( d_1 - \hat{\Sigma}\sqrt{T} \right), \\ d_3 &= (1 - \phi) \left( \frac{\beta_1 \ln\left(\frac{V^\infty}{V_1}\right) - (r + \frac{1}{2}\beta_1^2\Sigma_0^2)T}{\Sigma_0\beta_1\sqrt{T}} \right) + \phi \left( \frac{\gamma_1 \ln\left(\frac{V^\infty}{V_1}\right) - (\Omega_1 + \frac{1}{2}\hat{\Sigma}^2\gamma_1^2)T}{\hat{\Sigma}\gamma_1\sqrt{T}} \right), \\ d_4 &= (1 - \phi) \left( \frac{\beta_1 \ln\left(\frac{\bar{V}}{V_1}\right) - (r + \frac{1}{2}\beta_1^2\Sigma_0^2)T}{\Sigma_0\beta_1\sqrt{T}} \right) + \phi \left( \frac{\gamma_1 \ln\left(\frac{\bar{V}}{V_1}\right) - (\Omega_1 + \frac{1}{2}\hat{\Sigma}^2\gamma_1^2)T}{\hat{\Sigma}\gamma_1\sqrt{T}} \right), \\ d_5 &= (1 - \phi) \left( \frac{\ln\left(\frac{V_1}{V}\right) + (r - \delta - \frac{1}{2}\Sigma_0^2)T}{\Sigma_0\sqrt{T}} \right) + \phi \left( \frac{\ln\left(\frac{V_1}{V}\right) + (r - \delta - \frac{1}{2}\hat{\Sigma}^2)T}{\hat{\Sigma}\sqrt{T}} \right). \end{aligned}$$

**Proof 2** *See Appendix.*

As noted earlier, foreign direct investment is a path dependent process, i.e. expansion may be interpreted as a sequence of investments where each investment feeds back information that can be used to improve the quality of subsequent decisions. Hence, Proposition 2

states that while paying the initial costs  $I_1$  the firm obtains in return a stake in the IJV with value  $\epsilon V$  and the option to accrue further potential growth. Put differently, the initiated project serves as a platform for a second investment opportunity which is accessible once a preassigned collaboration period has passed. The firm will, however, only opt for this additional flexibility if the value of the option exceeds the corresponding costs  $\bar{I}$ . Again, we are able to assign an optimal threshold to this decision.

**Proposition 3** *The firm aims at buying out the host country partner, if the IJV's asset value  $V(t)$  at  $t_2$  reaches at least an optimal trigger value  $\bar{V}$  determined by:*

$$f(\bar{V}) - \bar{I} \stackrel{!}{=} 0. \quad (10)$$

*with  $\bar{I}$  as negotiation costs and  $f(V)$  as the value of the buyout option given by:*

$$f(V) = AV^{\beta_1} H(V \leq V^\infty) + ((\bar{\epsilon} - \epsilon)V - I_2) H(V > V^\infty). \quad (11)$$

Here  $H(\dots)$  denotes the Heaviside function which is equal to one if the condition expressed is fulfilled and zero otherwise.

**Proof 3** *See Appendix.*

Proposition 3 indicates that possessing the right to buy out the partner involves some negotiation costs. These costs comprise all costs required to set up a buyout option clause in the contract and hence represent the value of this right from the partner's perspective.<sup>22</sup> Consequently, only if the value of the option to buy out the partner exceeds these costs will the firm prefer a buyout option with the right to terminate the IJV at some optimal time in the future. On the opposite, the IJV will remain stable if the IJV's current value

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<sup>22</sup> See e.g. Chi and Seth (2002) or Chi (2000) for a further discussion on divergent valuation expectation in the context of IJVs.

at  $t_2$  does not exceed the optimal threshold. Hence we do not observe buyout clauses to become effective because they are too costly.<sup>23</sup>

Optimal termination of the IJV is conditional on the future development of the IJV's asset value  $V(t)$ . More precisely, the buyout option grants the right to acquire the remaining equity stake, i.e.  $(\bar{\epsilon}-\epsilon)$ , in exchange for the assigned costs  $I_2$ .<sup>24</sup> Consequently, the following proposition specifies when it is optimal for the firm to perform the buyout.

**Proposition 4** *The firm will switch from an IJV to a cross-border acquisition, i.e. buy out the host country partner, if the IJV's asset value  $V(t)$  reaches at least an optimal trigger value  $V^\infty$  determined by:*

$$V^\infty = (1 - \phi) \left( \frac{1}{(\bar{\epsilon} - \epsilon)} \frac{\beta_1}{\beta_1 - 1} I_2 \right) + \phi \left( \frac{1}{(\bar{\epsilon} - \epsilon)} \frac{\gamma_1}{\gamma_1 - 1} I_2 \right), \quad (12)$$

with  $\gamma_1 = \frac{1}{2} - (r - \delta)/\Sigma_1^2 + \sqrt{((r - \delta)/\Sigma_1^2 - 0.5)^2 + 2r/(\Sigma_1^2)}$  and  $\beta_1 = \frac{1}{2} - (r - \delta)/\Sigma_0^2 + \sqrt{((r - \delta)/\Sigma_0^2 - 0.5)^2 + 2r/(\Sigma_0^2)}$ .

**Proof 4** *See Appendix.*

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<sup>23</sup> We have implicitly assumed that once the firm has initiated the IJV it uses the collaboration period  $T$  to gather further information whether a buyout option seems economically advantageous or not. Consequently, a possible buyout clause will not be discarded initially. Rather, the firm makes their realization dependent on the future value development of the IJV. However, it is worthwhile stating that charging such costs may represent only a theoretically viable solution, in particular, because of the difficulties associated with getting the partner to agree on it. We thank an anonymous referee for pointing this out.

<sup>24</sup> It is assumed that the acquisition price is fixed right from the start. For a justification of this assumption refer to e.g. Beamish and Banks (1987) or Chi and McGuire (1996).

## 5 Comparative-static Analysis

This section presents a summarization of a comparative-static analysis of the derived individual stage trigger points. Unless noted otherwise, we will assume the following values  $I_1 = I_2 = 1$ ,  $\bar{I} = 0.5$ ,  $r = 0.05$ ,  $\delta = 0.03$ ,  $\rho = 0.5$ , and  $\bar{\epsilon} = 1$ . Allowing for a collaboration period of seven years, i.e.  $T = 7$ , the value of the flexibility the firm has to consider while planning to implement the IJV will be discussed first.

### 5.1 No Uncertainty Resolution

At date  $t = 0$  the firm has the option to initiate the IJV  $t_1$  years from now. The corresponding value is a convex function of the underlying asset value. Further, the option value is positively affected by the length of time the firm can defer the decision to initiate the IJV. This effect is more pronounced for high uncertainties which themselves affect the option value in a positive way, i.e. high uncertainties causes the option value to increase. Here, like for all other cases where the impact of uncertainty is discussed, it is important to keep in mind that it is not the uncertainty *per se* that makes an option valuable. Rather, it is the potential for improving managerial decisions based upon the availability of new information that affects the option value positively. Thus, the potential due to protection is the more valuable the greater the uncertainty. Where the initial equity share of the IJV is concerned the results depict that an increase in  $\epsilon$  increases the overall option value. Because the value of flexibility is not only driven by uncertainty but by irreversibility, too, the findings show that an increase in the initial set-up costs decreases the option value. Likewise, an increase in the cost of performing the buyout has a similar effect. However, its influence is less significant than that of the initial costs. Figure 1 summarizes the findings.



===== [INSERT FIGURE 1 HERE] =====

At time  $t_1$  the firm receives an IJV with value  $(\bar{\epsilon} - \epsilon)V - I_0 + F(V)$ . The option component  $F(V)$  comprises the subsequent possibility of buying out the partner in the host country. As indicated by Proposition 2 the firm will initiate the IJV at date  $t = t_1$  if  $(\bar{\epsilon} - \epsilon)V - I_1 + F(V)$  is greater than zero. Consequently, an optimal threshold exists that is not only driven by the immediate effect but also by the influence of subsequent possibilities, i.e. buying out the partner at a future date. Figure 2 depicts the sensitivity of the optimal threshold  $V^*$  at date  $t = t_1$  with respect to uncertainty.

===== [INSERT FIGURE 2 HERE] =====

As the results show, the threshold decreases as uncertainty increases. Further, an increase in the size of the initial equity share lowers the critical threshold and thus makes entry more likely. So why are minority IJVs then attractive? The answer lies in the interaction between equity and uncertainty. A low initial equity share corresponds to a high upside potential and due to the insurance effect, i.e. the downside risk constraint due to the option feature, the attractiveness of this subsequent flexibility is positively affected by uncertainty. Thus, the enhancing effect of uncertainty is the more distinct the lower the initial equity share. For instance, the critical value for majority IJVs is half the threshold for minority IJVs if the situation is evaluated under no uncertainty (see Figure 2). This situation reflects the classical net present value (NPV) setting. If uncertainty increases, however, this proportion changes up to 85 percent indicating that an increase in  $\sigma$  compensates the immediate value gains majority JVs generate. Put differently, high uncertainty makes the initiation of minority JV more likely. Further, without the impact of knowledge accumulation and learning the results indicate that the length of collaboration has only a minor impact on changes in the threshold and option value  $F(V)$  respectively. Finally, the costs associated with each stage increase the critical threshold.

From the standard literature the comparative static results for a perpetual call option  $f(V)$  are commonly known and therefore summarized briefly. We will discuss the consequences with respect to Proposition 3 first. As Proposition 3 designates, the firm has to decide at time  $t_2$  whether or not it opts for a buyout option. Consequently, for any IJV value  $V(t_2)$  less than  $\bar{V}$  we do not observe buyout clauses to become effective. Notably, the critical threshold  $\bar{V}$  shows the same properties as  $V^*$ , i.e. it decreases as overall uncertainty increases indicating an increased probability of IJVs' instability. This is because high uncertainty again raises the attractiveness of the subsequent investment  $f(V)$ . *Ceteris paribus*, a higher cost for this insurance, i.e.  $\bar{I}$  has a countervailing impact and decreases the likelihood of subsequent termination. Once the firm has decided in favor for a subsequent buyout, Proposition 4 denotes the condition for optimal termination of the IJV. Here, the attractiveness of exercising the option is reflected in the optimal threshold  $V^\infty$ . Hence, higher uncertainties imply higher threshold values which implicitly suggests that the buyout option is kept alive longer than for less uncertain asset values. Moreover, the size of initial equity share has a reciprocal influence on the threshold. High initial stakes increase the threshold and this effect becomes more pronounced the higher the uncertainty. Likewise, if the IJV is formed in host countries that restrict foreign ownership, i.e.  $\bar{\epsilon} < 1$  then the attractiveness of the buyout option is impaired and hence decreases the value of subsequent flexibility  $F(V)$ . Consequently, this provokes an increase of the critical market entry threshold  $V^*$  and lowers *ceteris paribus* the probability to initiate the JV. However, there is another effect attained to the restriction of foreign equity which is related to the stability of JVs. In particular, firms that have decided to buy the remaining shares will *ceteris paribus* show a greater propensity to wait. Consequently, we can expect that JVs in countries that impose a maximum share will be more stable than in countries where full acquisition is possible.

Up to now we have implicitly assumed that the exchange rate movement is positively correlated with the development of the IJV's value, i.e.  $\rho > 0$ . Consequently, an increase in the exchange rate volatility increases the overall uncertainty  $\Sigma$  and hence increases  $V^\infty$ . Put differently, the buyout strategy is further postponed. At the same time, however, the buyout option becomes more attractive so that the option value contributes to a greater extent to the compensation of the initial sunk costs which provokes a further decrease of the market entry threshold  $V^*$ . If, however, the correlation is negative then the impact of  $\sigma_E$  on the market entry threshold  $V^*$  and duration, as proxied by  $V^\infty$  becomes ambiguous. This is because  $sgn(\partial\Sigma/\partial\sigma_0)$  depends on whether  $\sigma_0 > 0.5\rho\sigma_E$  or not. In particular, for  $\sigma_0 > 0.5\rho\sigma_E$  the overall impact of an increase in  $\sigma_0$  is negative, i.e. an increase in  $\sigma_0$  increases the overall uncertainty related to subsequent investment opportunities and lowers  $V^*$  respectively. Contrary, if  $\sigma_0 < 0.5\rho\sigma_E$  then any increase in  $\sigma_0$  increases  $V^*$  because here, the overall uncertainty is reduced and the subsequent option becomes less attractive. Consequently, market entry becomes less likely, too (Figure 2).

## 5.2 *Uncertainty Resolution*

So far we have neglected the possibility that the firm can resolve uncertainty during co-operation by accumulating knowledge and learning from the host-country partner. Consequently, as it has been depicted, the length of the minimum collaboration period has no significant influence on the option value and the critical threshold for market entry. However, it seems unreasonable to generalize on these findings since the firm can mitigate risk by relying on local partners' resources, including their local knowledge, relationships with the government, or by experience with new capabilities (see, e.g. Inkpen and Beamish 1997). Consequently, it is reasonable to assume that this will have an impact on the overall uncertainty of the cross-border activities. In the following we will therefore summarize

the findings if uncertainty is allowed to be time-varying and furthermore decreases over time.

The corresponding value  $W(V)$  is again a convex function of the underlying asset value. With respect to the comparative static analysis, it shows almost the same properties, e.g. it increases with time to maturity and decreases with initial cost. However, the subsequent ability to resolve uncertainty due to cooperation causes some distinctive differences especially with respect to the uncertainty-sunk cost interdependence which are best discussed by focusing on the optimal threshold value indicating optimal market entry, i.e.  $V^*$ . Like the threshold for the basic scenario, the critical threshold for initiating the IJV is a decreasing function of the uncertainty. However, as opposed to the basic model, the threshold is generally larger indicating a lower propensity to engage in an IJV. This is due to the fact that the ability to learn lowers the subsequent option value  $F(V)$  and provokes an increase in the critical threshold. This effect is further amplified the quicker the firm can assimilate new knowledge, hence resolving uncertainty (see Figure 3). In addition, the threshold is now also highly sensitive to the length of  $T$ . An increase in  $T$  increases the option value, holding the decay factor fixed. This is due to the fact that the half-life period, i.e. the time span elapsed until the initial uncertainty is cut into half, is a function of the minimum collaboration period. Hence, longer half-time periods mean less uncertainty resolution which adds value to the option and lowers the critical threshold respectively.

===== [INSERT FIGURE 3 HERE] =====

Recent empirical results have disproved the assumption that IJVs in emerging economies carry a higher growth option value than those initiated in developed countries (see, e.g. Tong et al. 2008). Given the context of the paper, this can be explained by the impact of uncertainty resolution on growth option value  $F(V)$ . IJVs in emerging economies are more

challenged by effective knowledge accumulation and learning about the host country than those IJVs set up in mature industrialized countries with well advanced market-supporting institutions. Consequently, the decay factor is more pronounced once an IJV is set up in an emerging economy and hence lowers the value of subsequent expansion.

As can further be drawn from Figure 3, the threshold of the baseline model -given  $\sigma_0 \cong 0.27$ - equals the threshold under learning capabilities at  $\sigma_0 = 0.45$  (assuming a 50% reduction of uncertainty due to learning during  $T$ , i.e.  $\sigma_1 = 0.5\sigma_0$ ). Hence, given this example IJVs which foster learning have a similar risk-free entry probability when initiated in a more uncertain environment as IJVs initiated in environments with lower uncertainties, indicating the risk compensating effect of learning capabilities. Another interesting result stands out that addresses the question regarding an optimal equity choice. In particular, how do foreign firms structure their equity investments depending on their learning abilities? Given a similar uncertain environment, e.g.  $\sigma_0=0.27$ , we can determine the equity share that equates both thresholds. As Figure 3 depicts the firm can raise their initial equity share from 25% to 40%, *ceteris paribus*, if they expect greater learning gains from the IJV. Moreover, the higher the uncertainty the greater the incentive for the firm to opt for higher initial equity shares.

Finally, learning also impacts the overall stability of IJVs. A decrease from  $\sigma_0$  to  $\sigma_1$  lowers the buyout option value  $f(V)$ . Concurrently, the critical threshold  $V^\infty$  is lowered, too, indicating a higher propensity to initiate the buyout under a learning and knowledge accumulation framework. Consequently, if uncertainty can be resolved, the propensity to exercise the option subsequently, i.e. buy out the partner, increases. As more recently demonstrated, these results are in line with actual findings that the resolution of uncertainty surrounding alliances can lead firms to acquire additional equity from their partners (see, e.g. Kogut and Singh 1988, Folta and Miller 2002).

Given the endogenous derived thresholds we can deduce information on the stability of IJVs. As mentioned previously, the firm has to decide at date  $t = t_2$  whether or not it intends a termination of the IJV by means of a buyout. Thenceforward, termination is performed once the corresponding threshold is reached. We can express the first hitting times  $\tilde{t}$  formally as  $\tilde{t}_B = \inf\{t \geq 0; V(t) \geq V_B^\infty\}$  and  $\tilde{t}_L = \inf\{t \geq 0; V(t) \geq V_L^\infty\}$ , where  $B$  and  $L$  denote the basic and learning case. While these stopping times are themselves random variables, it is possible to estimate their expected value, i.e.  $\mathbb{E}[\tilde{t}]$ . For geometric Brownian motion, we get:

$$\mathbb{E}[\tilde{t}_B] = Z^{-1} \ln \left( \frac{V_B^\infty}{V(t_2)} \right), \quad (13)$$

$$\mathbb{E}[\tilde{t}_L] = Z^{-1} \ln \left( \frac{V_L^\infty}{V(t_2)} \right), \quad (14)$$

with  $Z = (r - \delta) - 0.5\Sigma^2 > 0$ .<sup>25</sup> If the condition  $Z > 0$  does not hold, the termination is never optimal and the collaboration remains active. Consider, for example, an environment where the initial equity equals 0.35 and overall uncertainty equals 0.1. Assuming  $V^*$  as the best predictor for the value at date  $t_2$ , i.e.  $\mathbb{E}[e^{-rT}V(t_2)] = V^*$ , we can specify the expected overall duration, i.e.  $T + \mathbb{E}[\tilde{t}_B]$ , of a collaboration as being 10 years before the firm will acquire the remaining stakes.<sup>26</sup> Ceteris paribus, IJVs with the ability to acquire and process knowledge, i.e. given an uncertainty resolvment of 10%, will observe an expected overall duration of 7.7 years.

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<sup>25</sup> For a detailed discussion on optimal stopping times for perpetual options, see Wilmott, Dewynne, and Howison (1993, p. 368ff.).

<sup>26</sup> The parameters used are:  $r = 0.05$ ,  $\delta = 0.03$ ,  $I = 1$ ,  $I_0 = 1$ ,  $\bar{I} = 0.25$ , and  $T = 7$ . Given these parameters the critical market entry threshold is above the optimal trigger value  $\bar{V}$  indicating a preference for subsequent buyout in all cases.

We can generalize on these findings by specifying the optimal stopping times as a function of initial equity share and environmental uncertainty. As Figure 4 depicts region 1 and region 2 are characterized by stable IJVs. In particular, region 1 designates equity-uncertainty combinations for which the buyout option is less attractive to the foreign investor. Hence, those IJVs remain stable by definition. Similar, region 2 indicates IJVs where the foreign investor has chosen to buy out the partner, however, optimal exercise of the real option exceeds 20 years and in some cases it is even never optimal to exercise the option. Contrary, region 3 characterizes IJVs that are less stable, i.e. their life span is shorter than 20 years. Given a low uncertain environment the results show that minority IJVs are less stable than majority IJVs. However, as uncertainty increases minority IJVs become more stable, too. Put differently, while uncertainty increases waiting for new information becomes more valuable even for these low resource commitments, i.e. low sunk costs. Hence it pays to profit from the insurance effect by keeping the buyout option alive. A similar effect is observed at the intersection of region 2 and 1. Given no uncertainty, the buyout option is less valuable to the firm if it possesses a majority stake. With the increase of uncertainty, however, the value of the buyout option  $f(V)$  becomes more attractive which is reflected in a decrease of the critical threshold  $\bar{V}$ . Consequently, the majority-owned IJV will become less stable.

With respect to the impact of learning we find that learning has an ambiguous effect on stability. More precisely, while majority IJVs become more stable, i.e. depicted by an extension region 1, minority IJVs become less stable. Here, we see that region 3 expands toward higher equity-uncertainty combinations. Consequently, buyouts will be observed more frequently. Interestingly, increased learning capabilities enhance these effects. Figure 4 summarizes the findings.

===== [INSERT FIGURE 4 HERE] =====

It is commonly agreed that the expansion abroad is a path dependent process which is reflected in the fact that the observed internationalization processes of firms change gradually over time. However, there is a gap in recent literature with respect to modeling the foreign market entry policies of firms. This model is a first attempt to stress the sequential nature of IJVs and to depict the importance of subsequent investment options on the initial entry decision and their effect on JV duration. Three sets of contributions emerge from the presented model. First, we derive a first rigorous model of the evolutionary sequence of a JV given subsequent possibilities to grow. It is demonstrated that for each stage there is a threshold value at which it is optimal for a firm to exercise the corresponding real option. The results show the new complementary insight that the decision to invest in the first stage is not only driven by the value of waiting, as commonly modeled in the literature, but is also driven by the flexibility to buy out, i.e. by exercising a subsequent growth option. Thus it alludes to the tensions between the impact of waiting and those value contributions that stem from growth options. In line with the empirical literature this aspect becomes crucial when high uncertainty, e.g. due to additional exchange rate risk, persists and if minority-owned IJVs are considered. Second, while recent literature has neglected the impact of uncertainty resolution this paper presents a first attempt to implement features of a time-varying uncertainty in a real option induced IJV setting. In particular, neglecting learning effects results in an overvaluation of the IJV due to an overemphasis on irreversible subsequent growth opportunities. Putting it differently, IJVs initiated for the purpose of knowledge accumulation and learning profit to a lesser extent from subsequent managerial flexibility. Contrary, being less exposed to uncertainty due to learning justifies to invest in higher equity shares at the outset. Moreover, the greater the ability to absorb new knowledge, the greater the propensity to buy out the partner.



Hence, learning capabilities lead on average to less stable IJVs. Third, recommendations for managerial actions of international operating firms can be deduced from the study. In particular, less explicit option clauses have been observed in the IJV context due to problems involving real option complexities and their valuations. Thus, the derived closed form solutions can help managers to structure and deploy these explicit option clauses more efficiently. Further, managers can use the presented structural option attributes and their corresponding valuation as an argumentative backing of certain - in the light of NPV based valuation - critical investment decisions.

While this model is subject to several limitations it points out some promising avenues for future research. In the absence of explicit call option clauses, the real option become implicit and potentially non-proprietary. Consequently, exercising these options at a later stage is subject to negotiation and in turn, additional cost. One way to deal with this would be to treat the subsequent costs as uncertain. Further, foreign market entry is not only a unidirectional path. In particular, firms might leapfrog certain entry modes or withdraw from the foreign market at a later date. An example of such a divestiture is the abandonment of a JV between Hitachi Ltd. and Texas Instruments Incorporated. Consequently, instead of acquiring the remaining shares from the partner, a divestment is a serious alternative for some internationally active companies. Moreover, today's collaborations are to a large extent driven by research and development. Hence, modeling the effect of technological uncertainty in a collaboration would be a fruitful way to extend the model. Apart from this, the model can be extended to account for tax rates or subsidies, leading to a debate concerning the implications for governmental policies to attract FDI. Finally, we believe that the present study provides new opportunities for further empirical research under an option framework.

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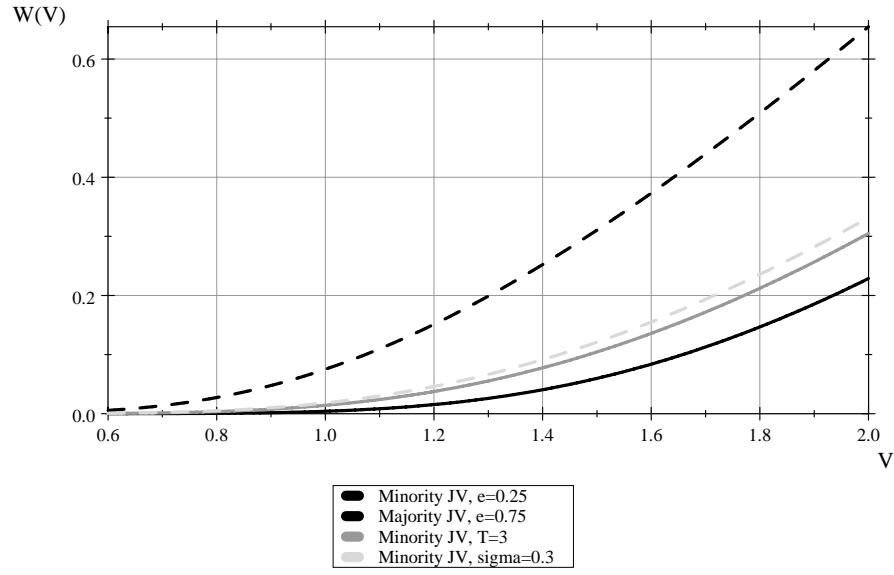


Figure 1. Value of pending cross-border joint venture  $W(V)$ . The parameters used are:  $r = 0.05$ ,  $\delta = 0.03$ ,  $\sigma = 0.27$ ,  $\sigma_E = 0.2$ ,  $\rho = 0.5$ ,  $I_0 = 1$ ,  $I_1 = 1$ ,  $\bar{I} = 0.5$ ,  $T = 7$ .

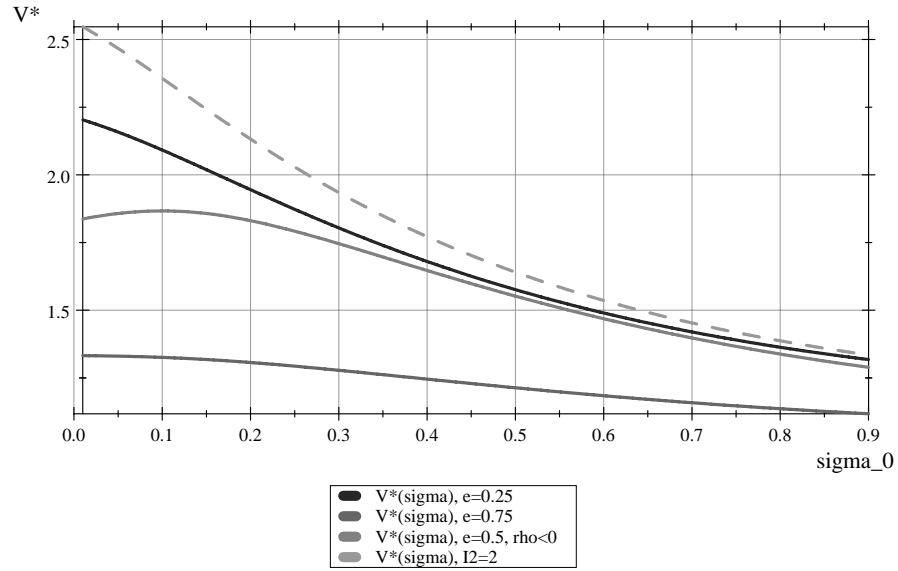


Figure 2. Optimal threshold  $V^*$  for initiating an international joint venture under environmental uncertainty. The parameters used are:  $r = 0.05$ ,  $\delta = 0.03$ ,  $\sigma_E = 0.2$ ,  $\rho = 0.5$ ,  $I_0 = 1$ ,  $I_1 = 1$ ,  $\bar{I} = 0.5$ ,  $T = 7$ .

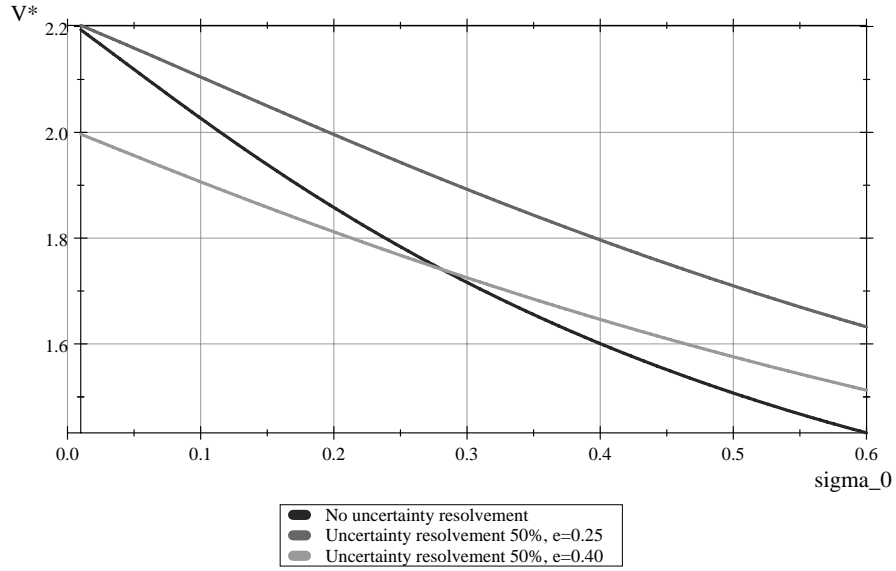


Figure 3. Difference in optimal thresholds  $V_L^*$  and  $V_B^*$  with respect to uncertainty and initial equity share. (L indicates the added learning feature while B indicates the base case). The parameters used are:  $r = 0.05$ ,  $\delta = 0.03$ ,  $\sigma_E = 0.2$ ,  $\rho = 0.5$ ,  $I_0 = 1$ ,  $I_1 = 1$ ,  $\bar{I} = 0.5$ ,  $T = 7$ , and  $\sigma_1 = 0.5\sigma_0$ .

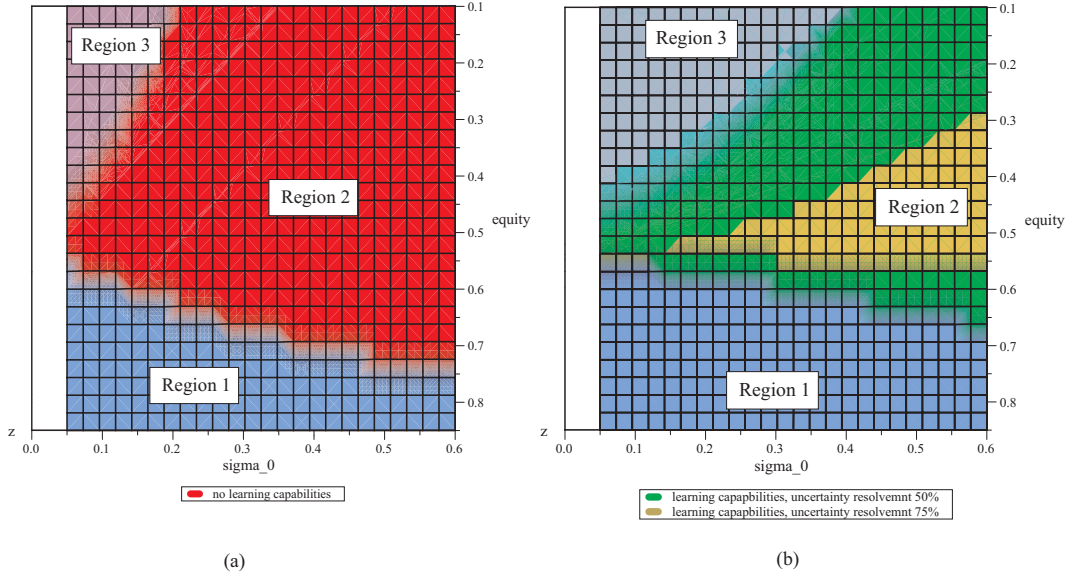


Figure 4. Expected Duration of IJVs with respect to initial equity share and uncertainty where (a) no learning capabilities (b) learning capabilities are present. The parameters used are:  $r = 0.05$ ,  $\delta = 0.03$ ,  $\sigma_E = 0.2$ ,  $\rho = 0.5$ ,  $I_1 = 2$ ,  $T = 7$ .



## 7 Appendix: Option Values and Investment Thresholds

The values of the investment opportunities  $W(V)$  and  $F(V)$ , as well as the optimal trigger point  $V^\infty$  representing the actual timing of the subsequent buyout may be solved recursively. First, the values and thresholds for the perpetual call, i.e.  $f$  and  $V^\infty$  have to be determined. Then the value of the second stage investment possibility  $F(V)$  and the corresponding trigger point  $V_1^*$  are derived. Finally, the value of the overall entry strategy  $W$  is specified.

### 7.1 Optimal Timing of Buyout Strategy

From the standard literature the results for a perpetual call option  $f(V)$  are commonly known.<sup>27</sup> Thus, they are just summarized briefly. Upon exercising the buyout option the firm retrieves the remaining shares which account for  $(\bar{\epsilon} - \epsilon)$  of the overall value of the IJV. Hence, the value for the perpetual call options results in:

$$f(V) = AV^{\beta_1} H(V \leq V^\infty) + ((\bar{\epsilon} - \epsilon)V - I_2) H(V > V^\infty), \quad (15)$$

with  $H(\dots)$  as the Heaviside function which is equal to one if the condition expressed is fulfilled and zero otherwise.  $A$  and  $\beta_1$  are the usual constants which are defined by:

$$\beta_1 = \frac{1}{2} - \frac{(r - \delta)}{\Sigma_0^2} + \left( \left[ \frac{(r - \delta)}{\Sigma_0^2} - \frac{1}{2} \right]^2 + \frac{2r}{\Sigma_0^2} \right)^{1/2}, \quad (16)$$

$$A = \left[ (\bar{\epsilon} - \epsilon) \frac{1}{\beta_1} \left[ \frac{1}{(\bar{\epsilon} - \epsilon)} \frac{\beta_1}{\beta_1 - 1} I \right]^{(1 - \beta_1)} \right]. \quad (17)$$

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<sup>27</sup> See, e.g. Dixit and Pindyck (1994).

The optimal trigger value  $V^\infty$  for the buyout strategy results in:

$$V^\infty = \frac{1}{(\bar{\epsilon} - \epsilon)} \frac{\beta_1}{\beta_1 - 1} I_2. \quad (18)$$

## 7.2 Closed-form Solution for the Collaboration Period

Referring to the above results, the value of the overall option during the collaboration period  $T = t_2 - t_1$  is given by:

$$F = E^Q \left[ \frac{[f(V) - \bar{I}]^+}{e^{r(t_2 - t_1)}} \right]. \quad (19)$$

This results in solving the following integrals:

$$\begin{aligned} F &= e^{-r(t_2 - t_1)} \left( \int_{\bar{V}}^{V^\infty} AV^{\beta_1} d\Phi(V) + \int_{V^\infty}^{\infty} ((\bar{\epsilon} - \epsilon)V - I_2) d\Phi(V) - \int_{\bar{V}}^{\infty} \bar{I} d\Phi(V) \right) \\ &= e^{-r(t_2 - t_1)} \left( \int_{-\infty}^{V^\infty} AV^{\beta_1} d\Phi(V) - \int_{-\infty}^{\bar{V}} AV^{\beta_1} d\Phi(V) + \int_{V^\infty}^{\infty} ((\bar{\epsilon} - \epsilon)V - I_2) d\Phi(V) - \int_{\bar{V}}^{\infty} \bar{I} d\Phi(V) \right), \end{aligned}$$

where  $d\Phi(V)$  denotes the implied probability measure. The last two integrals are similar to the Black-Scholes integrals and can be solved in the same manner. However, where the term in the middle is concerned, special attention is given to the  $V^\beta$  term. By applying Itô's Lemma  $dV^\beta$  and  $V^\beta$ , respectively, we obtain:

$$dV^\beta = (r - \delta)V^\beta dt + \Sigma_0 \beta V^\beta dB^Q, \quad (20)$$

$$V_T^\beta = V_0^\beta e^{rT - 1/2 \Sigma_0^2 \beta^2 T + \Sigma_0 \beta B_T^Q}. \quad (21)$$

The last two terms of the exponential function can be substituted by a stochastic process

$X \sim N(-1/2\Sigma_0^2\beta^2T, \Sigma_0^2\beta^2T)$ . Thus, the resulting integral

$$e^{-r(t_2-t_1)}A\left(\int_{-\infty}^{V^\infty}V^{\beta_1}d\Phi(V)-\int_{-\infty}^{\bar{V}}V^{\beta_1}d\Phi(V)\right), \quad (22)$$

can be simplified by substituting equation (21) into (22). Applying standard methods and substituting  $T = t_2 - t_1$  we obtain equation (9) for  $\phi = 0$ .

### 7.3 Closed-form Solution for the Compound Option

The solution of  $F$  is valid at time  $t_1$ . However, if we want to know the value of the compound option, we have to determine the value of this option at time  $t_0$ . Thus, one also has to solve:

$$W(V) = E^Q \left[ \frac{[\epsilon V + F(V) - I_1]^+}{e^{r(t_1-t_0)}} | \mathcal{F}_0 \right], \quad (23)$$

given the filtration  $\mathcal{F}_0$ . The solution procedure is similar to that provided by Geske (1979).

Setting  $t_0 = 0$ , one has to solve the following integral

$$W(V) = e^{-rt_1} \int_{V_1^*}^{\infty} (\epsilon V + F(V) - I_1) d\Phi(V), \quad (24)$$

with respect to the given solution of the foremost closed-form solution for  $F(V)$ , i.e.:

$$F(V) = (\bar{\epsilon} - \epsilon)V_1e^{-\delta T}N(d_1) - I_2e^{-rT}N(d_2) + AV_1^{\beta_1}(N(d_3) - N(d_4)) - \bar{I}e^{-rT}N(d_5). \quad (25)$$

The lower boundary  $V_1^*$  represents the threshold for exercising the compound option according to:<sup>28</sup>

$$\epsilon V_1^* + F(V_1^*) = I_1. \quad (26)$$

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<sup>28</sup> See e.g. Geske (1979).

By making use of the definition for the bivariate cumulative standard normal distribution

$M(\dots)$ , i.e.:

$$M(x, y; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^y \int_{-\infty}^x e^{-\frac{1}{2} \frac{(x^2 - 2\rho xy + y^2)}{1-\rho^2}} dx dy, \quad (27)$$

and the substitution  $\rho = \sqrt{t_1/t_2}$  the solution of the seven integrals is straightforward and leads to the expression of the value of the compound option  $W(V)$  for  $\phi = 0$  (see equation (7)).

#### 7.4 Learning

In the following, the solution is sketched for situations where uncertainty resolution takes place. It is assumed that cooperative actions in the second phase lower the asset value uncertainty  $\sigma$ . Further, learning does not affect the exchange rate uncertainty  $\sigma_E$ . Hence, the dynamics of  $V(t)$  change to:

$$dV/V = (r - \delta)dt + \Sigma^2(t)dZ^Q, \quad (28)$$

where

$$\Sigma^2(t) = \sigma^2(t) + \sigma_E^2 + 2\sigma(t)\sigma_E\rho. \quad (29)$$

Knowledge accumulation is exponential over time which is described formally by:

$$\sigma^2(t) = \sigma_0^2 e^{\left(\frac{2\ln(\sigma_1/\sigma_0)}{T}(t-t_1)\right)}, \quad (30)$$

Consequently, a decrease in uncertainty from  $\sigma_0$  to  $\sigma_1$  corresponds with a decrease of the overall uncertainty from  $\Sigma_0$  to  $\Sigma_1$ . By means of the Itô integral a solution can be derived

for the terminal asset value  $V(t)$  at time  $T$  expressed in domestic currency which results to:

$$\begin{aligned} V(T) &= V_0 e^{(r-\delta-\frac{1}{2}\left(\frac{1}{t_2-t_1} \int_{t_1}^{t_2} \Sigma^2(\tau) d\tau\right)T + \left(\frac{1}{t_2-t_1} \int_{t_1}^{t_2} \Sigma^2(\tau) d\tau\right)\sqrt{T}Z_{N(0,1)}} \\ &= V_0 e^{(r-\delta-\frac{1}{2}\hat{\Sigma}^2)T + \hat{\Sigma}\sqrt{T}Z_{N(0,1)}}, \end{aligned} \quad (31)$$

with

$$\hat{\Sigma}^2 = \frac{1}{2} \frac{(\sigma_1^2 - \sigma_0^2)}{\ln(\sigma_1/\sigma_0)} + \sigma_E^2 + \frac{2\sigma_E(\sigma_1 - \sigma_0)\rho}{\ln(\sigma_1/\sigma_0)}. \quad (32)$$

#### 7.4.1 The Buyout Option

When the MNE enters the period where it can trigger the buyout, the corresponding uncertainty is  $\sigma_1$ . Thus, the buyout option results in:

$$f(V) = BV^{\beta_1} H(V < V^\infty) + ((\bar{\epsilon} - \epsilon)V - I_2) H(V > V^\infty), \quad (33)$$

with

$$\begin{aligned} B &= \left[ (\bar{\epsilon} - \epsilon) \frac{1}{\gamma_1} \left[ \frac{1}{(\bar{\epsilon} - \epsilon)} \frac{\gamma_1}{\gamma_1 - 1} I \right]^{(1-\gamma_1)} \right], \\ \gamma_1 &= \frac{1}{2} - \frac{(r - \delta)}{\Sigma_1^2} + \left( \left[ \frac{(r - \delta)}{\Sigma_1^2} - \frac{1}{2} \right]^2 + \frac{2r}{\Sigma_1^2} \right)^{1/2}, \end{aligned} \quad (34)$$

and

$$V^\infty = \frac{1}{(\bar{\epsilon} - \epsilon)} \frac{\gamma_1}{\gamma_1 - 1} I_2. \quad (35)$$

#### 7.4.2 The Learning Phase

As discussed beforehand, this period is characterized by a time-varying decrease in uncertainty. Referring to the above results, the value of the overall option during the collab-

oration period  $T = t_2 - t_1$  is given by:

$$F(V) = E^Q \left[ \frac{[f(V) - \bar{I}]^+}{e^{r(t_2 - t_1)}} \right]. \quad (36)$$

with  $V(t)$  given by equation (31). Applying the standard risk-neutral pricing technique and substituting:

$$\Omega_1 = \frac{1}{2} \gamma_1 \hat{\Sigma}^2 (\gamma_1 - 1) + (r - \delta) \gamma_1.$$

the solution of the flexibility  $F(V)$  at date  $t_1$  is derived for  $\phi = 1$  (see equation (9)).

#### 7.4.3 The Waiting Period

During this stage, the uncertainty is at its initial level of  $\sigma_0$  and  $\Sigma_0$  respectively. Consequently, the value of managerial flexibility at time  $t_0$  can be determined by solving:

$$W(V) = E^Q \left[ \frac{[\epsilon V + F(V) - I_1]^+}{e^{r(t_1 - t_0)}} | \mathcal{F}_0 \right], \quad (37)$$

given the filtration  $\mathcal{F}_0$ . Substituting equation (9) for  $\phi = 1$  into (37) leads to an expression with seven integrals. The solution procedure is similar to the one described in 7.3. Applying the following substitutions:

$$\hat{\rho} = \Gamma \sqrt{t_1/t_2}, \quad (38)$$

$$\Gamma = \frac{\Sigma_0}{\hat{\Sigma} \sqrt{1 + ((\Sigma_0/\hat{\Sigma})^2 - 1)(t_1/t_2)}}, \quad (39)$$

$$\Omega_2 = \frac{1}{2} \gamma_1 \Sigma_0^2 (\gamma_1 - 1) + (r - \delta) \gamma_1, \quad (40)$$

$$M(x, y; \rho) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \int_{-\infty}^y \int_{-\infty}^x e^{-\frac{1}{2} \frac{(x^2 - 2\rho xy - y^2)}{1 - \rho^2}} dx dy, \quad (41)$$

solving the integrals by parts leads to the value of the compound option  $W(V)$  for  $\phi = 1$  as illustrated by equation (7).

## 8 Mixed-Brownian Motion

Following the usual risk-neutral valuation framework, the diffusion processes of  $\tilde{V}$  and  $E$  follow the respective lognormal processes:

$$d\tilde{V}/V = (r_f - \delta - \sigma_0\sigma_E\rho)dt + \sigma_V dB, \quad (42)$$

$$dE/E = (r - r_f)dt + \sigma_E dZ. \quad (43)$$

where  $r$  and  $r_f$  are the constant domestic and foreign interest rates, respectively, and  $\delta$  is the constant dividend yield of the asset in the foreign currency. The corresponding volatilities  $\sigma_0$  and  $\sigma_E$  are constant and let  $\rho$  be the instantaneous correlation between the Wiener increments  $dB$  and  $dZ$ . Applying Itô's lemma, i.e.

$$dV = \frac{\partial V}{\partial \tilde{V}} d\tilde{V} + \frac{\partial V}{\partial E} dE + \frac{\partial V}{\partial \tilde{V} \partial E} dE d\tilde{V}, \quad (44)$$

the risk-neutral dynamics of  $V(t) \equiv E(t)\tilde{V}(t)$  will be

$$dV = (r - \delta)Vdt + \Sigma_0 V dB, \quad (45)$$

with  $\Sigma_0^2 = \sigma_0^2 + \sigma_E^2 + 2\sigma_0\sigma_E\rho$ . Note that the dynamics of  $d\tilde{V}$  have an additional drift factor. This is because we have changed the numeraire from the money market account in foreign currency to the money market account in domestic currencies (see, e.g. Hull 2009, p. 673ff.).