Toward a better understanding of Participation, Length and Position in Global Value Chains

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Abstract

In this article we propose a revised methodology of value chain analysis in international input-output framework. We demonstrate that existing measure of GVC participation share as well as most often used method of decomposition of upstream and downstream value chain length on domestic and global component systematically underestimate GVC share significantly, because of unrealistic assumptions and calculation deficiencies. Proposed new methodology solves conceptual inconsistencies of existing measures, better captures the real proportion of GVCs and improves their understanding. More comprehensive conceptual approach offers researchers new and improved tools applicable to various analyses. The main contribution of proposed methodology is conceptual: we treat the value chain as a holistic entity and integrate both downstream and upstream parts into one conceptual framework, as opposed to existing separate treatment of upstream and downstream parts in isolation.

Keywords: Global value chain, domestic value chain, value chain length, relative position.

1 Introduction

International economics is putting increasing focus on the analysis of the dynamic network structure of the global economy - the way economies are linked, specialize, and grow, which is partly reflected and partly caused by the way global value chains are structured (henceforth GVCs - a network of production sharing firms from at least two different countries). Since the joint action of international organisations to coordinate and harmonize national input-output data into international input-output framework (WIOD, OECD TiVA), international economics and international trade theory have become increasingly reliant on international input-output (I-O) methodology. It is based on analysis of trade and production sharing in value-added terms in contrast with the traditional trade data in gross terms. As opposed to analysis of isolated firm value chains, this enables research on aggregate sectoral levels, that can at least to a certain extent capture structural changes and evolution of trade and production sharing. The main goal of the article is to demonstrate conceptual problems of existing most widely used value chain measures and to propose conceptual as well as mathematical solution to detected inconsistencies by deriving a new methodology for value chain analysis, offering researchers various new concepts and measures applicable to diverse empirical designs.

Early I-O measures of GVC structure were simple upstream and downstream indicators corresponding to the measure of distance to final demand (upstream) and to the Leontief backward linkage measure (downstream indicator), often labelled as the length of a value chain (Ahmad *et al.*, 2017). Fally (2011)

and Antras (*et al.*, 2012) defined downstream indicator to "reflect how many plants (stages) are sequentially involved in the production" up to observed point and upstream indicator "to measure how many plants this product will go through (e.g. by being assembled with other products) before reaching final demand (Fally, 2011, 10)". Fally (2011) defined them as a number of vertical stages weighted by the value added of each stage and distance between each stage being set to 1.[2] From then on, the average vertical distance has been the basic measure of value chain length conceptualisation within international I-O framework. Miller and Temurshoev (2015) further clarified existing measures, by presenting upstream and downstream indicators in matrix formulation using Ghosh's forward and Leontief's backward coefficient matrices (Ghosh, 1958; Leontief, 1936). Upstream and downstream measures there are simple measures of upstream and downstream length of value chains as measured by average vertical distance. Within this framework further upgrades were introduced by Wang (*et al 2017*) and Muradov (2016) who focused on separation of domestic from global production component. They created most widely used accounting framework for GVC participation, length and position so far applied by most prominent research on GVC, conducted jointly by WTO, WB group, OECD, IDE-JETRO, RCGVC-UIBE, and the China Development Research Foundation (GVC development reports).

Remainder of the article is structured in the following way: in the section 2 we start by a review of existing measures, describe upgraded conceptual design and conceptually compare selected measures. In the section 3 we present derivation of proposed measures: participation shares, total length of value chains and relative position in value chains and include decomposition of transaction to final consumer. In the section 4 we present some basic empirical results of new methodology and insights based on new GVC indicators. We conclude by discussing contributions and future research.

2 The comparison of conceptual design of existing and new measures

Measures overviewed in previous section however still suffer from conceptual inconsistencies most likely linked to their path-dependent conceptual evolution. What follows is a discussion of conceptual changes that we propose with respect to currently used measures that attempt to solve detected inconsistencies and improve understanding of value chain measures.

1. Point of observation of GVC (usually a sector-country pair) is just a small part of the whole production within its value chain that spans downstream as well as upstream. The most problematic practice that accompanies GVC measures since their emergence, has been completely isolated calculation of downstream and upstream components, regardless of their use as a measure of GVC participation, length or relative position. At the aggregate level and taking into account all value chains - domestic as well as foreign - downstream and upstream lengths are legitimately grasped by two completely separate calculations, first by Leontief inverse ($\mathbf{1}^T(I-A)^{-1}$) and second by Ghosh inverse ($(I-G)^{-1}\mathbf{1}$). However, as soon as we distinguish between domestic and GVC components, isolated calculation of downstream and upstream parts yields conceptually erroneous results. For example when observing a value chain, even if downstream value chain linkages are completely domestic, if their forward linkages predominantly include also foreign production sharing, label of this downstream part as being completely domestic is misleading and false - since it is obviously predominantly part of GVC through its upstream linkages.

Contemporary measures simply cut the value chain at the point of observation, disregarding upstream

[2]Using similar method as for calculation of the average propagation length required for analysis of dynamic response to shocks defined by Dietzenbacher and Romero (2007)

value chain when observing downstream part and *vice versa*. Wang's accounting typology tries to remedy this problem, but takes into account only the first stage that follows in one-sided value chain disaggregation[3]. Observing only first stage[4] that follows point of observation is just a partial solution, which doesn't take into account the rest of the value chain that follows (downstream) or precedes it (upstream case) and also leads to unnecessary computational complications and complexity. Due to the fact that GVC measures are calculated by subtracting isolated domestic component, demonstrated inconsistencies lead to overestimation of domestic[5] and consequential underestimation of GVC component in terms of participation shares as well as GVC components of value chain length. They also lead to inconsistent results in terms of relative position measure, because it is based on measures of length.

Wang's as well as Muradov's framework conceptually treat value chains as being divided into two completely separated components: downstream and upstream. In this way these components are not integrated at the most important point - the sector-country pair that is being observed - which is also the only point which connects upstream and downstream value chain component into one observed unity. Our proposal is to evaluate the value chain as a whole, from its beginning (treated as primary producer's value added) across all the downstream value chain linkages which share production of intermediaries used by observed sector-country pair and are after production at the point of observation further disseminated across all of the upstream value chain linkages until all of the observed output reaches final demand. The legacy of existing analyses is that the concepts and related calculations come in pairs: downstream and upstream GVC participation rates, downstream and upstream GVC length, etc. On the contrary, our proposal introduces one concept that is able to describe the whole chain. Participation shares are disaggregated in such a way to take into account both upstream and downstream linkages and the concept of total value chain length is introduced to measure vertical fragmentation of the chain as a whole, while relative position captures all the information about relative upstream or downstreamness. Two one-sided disaggregation are merged into one, each part of downstream disaggregation taking into account the upstream structure and *vice versa*.

2. Most widely used relative position measures, apart from demonstrated miscalculation due to isolated analysis of upstream and downstream parts, also suffer from inefficient representation. The calculation of these measures as ratio between all upstream and all downstream stages of production informs readers only about ranking of sector-country pairs (more or less upstream). Expressing relative position as the ratio of upstream stages to both upstream plus downstream stages offers (all intermediate production sharing stages) gives the same information of ranking the sector-country pairs as previous approaches, in addition to the information about the actual share of downstream and upstream stages in the value chain (for example, 0.7 upstream relative position measure in proposed representation gives information that from the point of observation 70% of value chain is downstream and 30% upstream).

3. Analysis of value chains shouldn't focus exclusively on GVCs, but also on domestic value chains (henceforth DVCs), since their evolution and relation with GVCs are essential for understanding of the economy and the effects of globalisation. Existing disaggregation of participation shares on domestic component and GVCs consists of simple duality that should by construction sum to 1, offering little room for domestic value chain analysis and their relation to GVCs. If domestic component and GVCs are the

[3]Summarizing his accounting typology: when observing downstream chain, next stage is either domestic consumption, further domestic production or export for consumption or further foreign production, and *vice versa* for upstream analysis.

[4]And second for further disaggregation of GVCs on simple and complex GVCs based on whether only one cross-border transaction or more takes place.

[5]Overestimation comes from demonstrated conceptual problem: downstream purely domestic value chain which at least partly continues in upstream part through global connections is clearly overestimated if simply whole isolated downstream purely domestic part is taken as representation of purely domestic chain. Overestimation is due to labelling proportion which takes global character in upstream extension as domestic. If in the end (even after one or two domestic transactions) any of the value added is sold abroad for further production, it is conceptually wrong to label such downstream part purely domestic, despite being purely domestic within its downstream part.

only two value chain participation shares, increase in global component necessarily means decrease in domestic and *vice versa*. However, existing domestic component also includes part of output that has no value chain: one that is produced by observed sector and ends in consumption without any (even domestic) production linkages. Because existing separation of GVCs from domestic component fails to properly capture DVCs, we propose to further disaggregate the output used in calculation of participation shares. We divide existing domestic participation share into two parts: the share which involves no production sharing (henceforth NVC - no value chain production) and share which includes only domestic production sharing - DVC. This enables detailed examination of DVC evolution and complements GVC analysis, offering possibility to capture relations between GVCs and DVCs. Empirical relevance of proposed concept of DVC is already seen by observing world averages. While existing approach demonstrates growth of GVCs and decline of domestic component, our approach shows long term growth of both GVCs and DVCs (Figure 2).

4. Wang's disaggregation of GVCs to simple and complex uses is based on the number of cross border transactions, regardless if value passes border for production or is it a simple export to final consumers. With such a criterion two conceptually different transactions are mixed, which leads to unnecessary computational complexity as well as impossibility of further conceptual disaggregation. The framework of international I-O analysis enables separate analysis of final transaction to consumer and transactions between firms. Accordingly, we have disaggregated GVCs to simple and complex using a prime criterion of the number of cross border transactions between firms (production sharing). Further disaggregation based on final transaction is made *post festum* and for all categories separately. Simple GVC is defined as a value chain with one cross border production sharing transaction (only 2 countries are involved in production) while complex GVC denotes a value chain with 2 or more cross border production sharing transactions.

Such decomposition is a starting point for disaggregation of measures to those that correspond to value chains that end with final exporting or domestic final consumption. This enables a more detailed and comprehensive analysis by comparing relative position, length and participation share measures of GVCs (simple or complex) and DVCs conditional on the value chain ending with either export or domestic consumption.[6]

5. Most of the researchers using upstream and downstream measures claimed that their measures capture only snake structure and not the spider structure of GVC[7]. We consider and demonstrate that perception of value chains as snakes and spiders is oversimplified. Length of value chain is measured by the average number of vertical transactions, which implies that any information about horizontal fragmentation is lost. However, even if a firm is part of a value chain organized as a spider (which is in general true for all GVCs), measure of vertical fragmentation takes this into account and explores further vertical linkages across all different intermediate inputs of a spider structure. In this sense vertical fragmentation captures (downstream and upstream measures) more than just snake structures, it measures all the vertical component in the complexly interlinked international economy, regardless of them being separated in the form of a spider or a simple snake, which is rarely the case.

To further back up our argument, we propose a different metaphor for existing measures of vertical fragmentation. Observing a concrete sector-country pair, forward and backward vertical fragmentation in the international economy could be better represented by a fractal tree (Figure 1). The primary value added

[6]Despite using different criterion for simple and complex GVC disaggregation, proposed decomposition includes all the possible varieties of existing disaggregation and offers simple translation of measures using existing definition of simple and complex GVC into proposed measures.

[7]"My index only captures snakes and is indifferent to spiders. (Fally, 2011)" and "More accurately, the index is a measure of the average number of stages (plants) involved in the production chain, weighted by the value added at each stage, and this in turn presupposes that the production chain follows a sequential (snakes) rather than concurrent (spiders) process" (Ahmad *et al.*, 2017).

is represented by the structure of the fractal roots, where production is only partly created by direct input of labour in each stage since it requires intermediaries, which are again further decomposed in the same manner *ad infinitum*. Similarly, product is being partly consumed immediately after production, but also partly sent to different production stages. From each of this upstream stages it is further decomposed in the same way (*etc. ad infinitum*) which spreads like fractal branches and leaves until it completely ends in final consumption. Roots represent production sharing until the point of observed sector-country pair, while branches and leaves a path of already produced output towards final consumption that includes upstream production sharing. Both structures are mathematically similar to fractals since coefficients of high powers of matrices A and G are becoming increasingly similar in their proportion and also increasingly smaller in size, due to the fact that all the coefficients are between 0 and 1, thus achieving infinite complexity as well as self-similarity. Value chains are thus infinitely complex but finite two-sided asymmetrical fractal objects, specific for each sector-country pair.[8]

Methodology	Antras, Fally, Miller	Wang, Muradov	Knez
Separation of domestic from global	No	Yes	Yes
Value passing through observed sector taken into account (linking upstream and downstream)	No	No (only up to second stage)	Yes
Relative (upstream) position measure structure	Downstream and upstream length used as position measures	Downstream vs. upstream number of intermediate stages ratio	Downstream vs. downstream plus upstream number of transactions between firms ratio
Participation share disaggregation	No	Domestic component, simple GVC, complex GVC	No value chain, DVC, simple GVC and complex GVC
Length measurement unit interpretation	Vertical production stages	Vertical production stages	Vertical number of transactions

Table 1: Comparison of different GVC indices conceptualisation.

3 Derivation of new proposed measures

3.1 Notation

International input-output data comprises two main data structures: the block matrix of all intermediate production flows C and the block matrix of all final consumption F. The Leontief matrix of input-output coefficients (matrix A) and the Ghosh matrix of producer coefficients (matrix G) together represent the basic underlying structure for any input-output analysis. We further decompose the matrices A and G into parts which correspond to domestic and cross-border transfers ($A = A_{CB} + A_D$ and $G = G_{CB} + G_D$). Because of the block structure of the data set, the block diagonal matrices of C and thus also of A and G represent domestic transfers of value between firms, while off diagonal block elements represent all cross-border transfers of value between firms. $\mathbf{1}$ represents vectors of ones of $n * m$ dimension, where n and m are number of countries and sectors respectively, while $\vec{1}$ represents vector of ones of n dimension.

[8]Poetically we could say that economy is then a dense forest, where value chains are complexly intertwined, roots of one value chain also being part of others draining the dispersed labour effort, while trees of different heights are stretching up towards the sky with their leaves in "struggle" for limited sunlight - realisation with final consumption.

Matrices C, A, G and their domestic and cross border decomposition have $(n*m)\times(n*m)$ dimension, while matrix F has a $(n*m)\times n$ dimension.

3.2 Participation shares

First, we decompose the total output of each observed sector-country as a share of production which:
a) has no value chain (abbreviated No Value Chain share - $NVCs$);
b) is part of exclusively domestic value chain (abbreviated Domestic Value Chain share - $DVCs$);
c) is part of global value chain (abbreviated Global Chain share - $GVCs$), which can be further decomposed into simple ($SGVCs$) and complex GVC share ($CGVCs$).

At the outset we evaluate the following Schur product which can be decomposed into 2 by 2 structure:

$$\left\{\left[\mathbf{1}^T(I-A)\right]\left[(I-A_D)^{-1}\right]\right\}^T\odot\left\{(I-G_D)^{-1}\left[(I-G)\mathbf{1}\right]\right\}=$$

$$=\left\{\left[\mathbf{1}^T(I-A)\right]\left[I+(I-A_D)^{-1}A_D\right]\right\}^T\odot\left\{\left[I+(I-G_D)^{-1}G_D\right]\left[(I-G)\mathbf{1}\right]\right\}=$$

$$=\left[\mathbf{1}^T(I-A)\right]^T\odot\left[(I-G)\mathbf{1}\right]+$$

$$+\left[\mathbf{1}^T(I-A)\right]^T\odot\left\{\left[(I-G_D)^{-1}G_D\right]\left[(I-G)\mathbf{1}\right]\right\}+$$

$$+\left\{\left[\mathbf{1}^T(I-A)\right]\left[(I-A_D)^{-1}A_D\right]\right\}^T\odot\left[(I-G)\mathbf{1}\right]+$$

$$+\left\{\left[\mathbf{1}^T(I-A)\right]\left[(I-A_D)^{-1}A_D\right]\right\}^T\odot\left\{\left[(I-G_D)^{-1}G_D\right]\left[(I-G)\mathbf{1}\right]\right\}$$

The first element of Schur product decomposition represents a share of total output that doesn't involve any production sharing (exclusively orange part of the tree in Figure 1) and is represented by Schur product of value added share and final stage share vectors (abbreviated No Value Chain share - $NVCs$):

$$NVCs=\left[\mathbf{1}^T(I-A)\right]^T\odot\left[(I-G)\mathbf{1}\right]$$

Second element refers to observed sector's value added which is transferred through upstream domestic value chain; third element relates to downstream domestic value added which ends as observed sector's final stage share and fourth is downstream domestic value added that is used as intermediate in production in upstream domestic value chain until it reaches final demand.

All of these (2nd, 3rd and 4th) elements represent the share of total output that consists of only domestic production sharing and is thus part of purely domestic value chain. This can be summarized by the following equation:

$$DVCs=\left\{\left[\mathbf{1}^T(I-A)\right]\left[(I-A_D)^{-1}\right]\right\}^T\odot\left\{(I-G_D)^{-1}\left[(I-G)\mathbf{1}\right]\right\}-$$

$$-\left[\mathbf{1}^T(I-A)\right]^T\odot\left[(I-G)\mathbf{1}\right]$$

Because $NVCs$ and $DVCs$ together represent the share of output that is part of value chains that don't share production across borders, simply expressing $\mathbf{1}-NVCs-DVCs$ will represent the share of output

that is included in production that crosses border at least once.[9] In figure 1, all non-global parts of value chain are presented as exclusively red and orange combinations. The parts of value chain that include one or more foreign linkages (coloured black in the Figure 1) belong to global part of the value chain, even if foreign linkage is some vertical distance away and even if the rest of linkages are domestic (red and orange combinations with at least one black linkage). Global value chain share is thus simply:

$$GVCs = \mathbf{1} - \left\{ \left[\mathbf{1}^T(I - A)\right] \left[(I - A_D)^{-1}\right] \right\}^T \odot \left\{ (I - G_D)^{-1} \left[(I - G)\mathbf{1}\right] \right\}$$

In order to further disaggregate GVC share into complex and simple GVC share we establish a clear criterion for such division. If there is *only 1 cross-border transaction between firms*, we denote it *simple GVC*; if there are *2 or more cross-border transactions between firms*, we denote it as *complex GVC*. We express their shares by evaluating all the possible combinations of value chains that include exactly one cross-border transaction between firms that are represented by 2 elements:
(1.) share of output consisting of all downstream value chains with exactly one cross-border transaction which ends as respective sector's final stage share or is used in production in its purely domestic upstream value chain until it reaches final demand and
(2.) share of output consisting of all upstream value chains with exactly one cross-border transaction which begins as respective sector's primary value added or as its purely domestic downstream value chain. Together they represent the simple GVC share of total output:

$$SGVCs = \left\{ \left[\mathbf{1}^T(I - A)\right] \left[(I - A_D)^{-1} A_{CB} (I - A_D)^{-1}\right] \right\}^T \odot \left\{ (I - G_D)^{-1} \left[(I - G)\mathbf{1}\right] \right\} +$$

$$+ \left\{ \left[\mathbf{1}^T(I - A)\right] \left[(I - A_D)^{-1}\right] \right\}^T \odot \left\{ (I - G_D)^{-1} G_{CB} (I - G_D)^{-1} \left[(I - G)\mathbf{1}\right] \right\}$$

Complex GVC represents the rest of the GVC share:

$$CGVCSs = GVCs - SGVCs$$

We conclude presented disaggregation of value chain output into proposed participation shares of 4 distinctive categories:

$$NVCs + DVCs + SGVCs + CGVCs = \mathbf{1}$$

3.3 Length of value chains

3.3.1 General total value chain length

Our measure of vertical fragmentation is denoted as total value chain length L to differentiate it from existing measure called GVC length which corresponds to downstream measure D. We define total value chain length L as the number of all downstream and upstream vertical transactions plus 1, which represents a transaction to final consumer. [10]

$$L = \underbrace{[\mathbf{1}^T(I - A)^{-1}A]^T}_{\text{Downstream transactions between firms}} + \underbrace{[(I - G)^{-1}G]\mathbf{1}}_{\text{Upstream transactions between firms}} + \underbrace{\mathbf{1}}_{\text{Transaction to final consumer}}$$

[9] Using condition that at least two different countries are involved in production, be it downstream or upstream.

[10] All existing measures as well as proposed new methodology use the same basic measurement unit of value chain length, which is the value transferred across the value chain in relation to observed sector-country total output. So far the basic measurement unit of value chain length is in most cases interpreted as being either some abstract distance (Antras *et al.*, 2012, 413) or simply number of production stages (Fally, 2011; Miller and Temurshoev, 2015). However, we believe that interpretation of basic unit of value chain length is better interpreted as *the average number of vertical transactions* as opposed to vertical stages. This interpretation (computationally equivalent to measuring production stages) helps to clarify the treatment of primary production stages and final production stages, which with our interpretation turn into transaction to final consumer (final stage) and payments of wages, profits and rents (primary stage). Both of those transactions in terms of total output

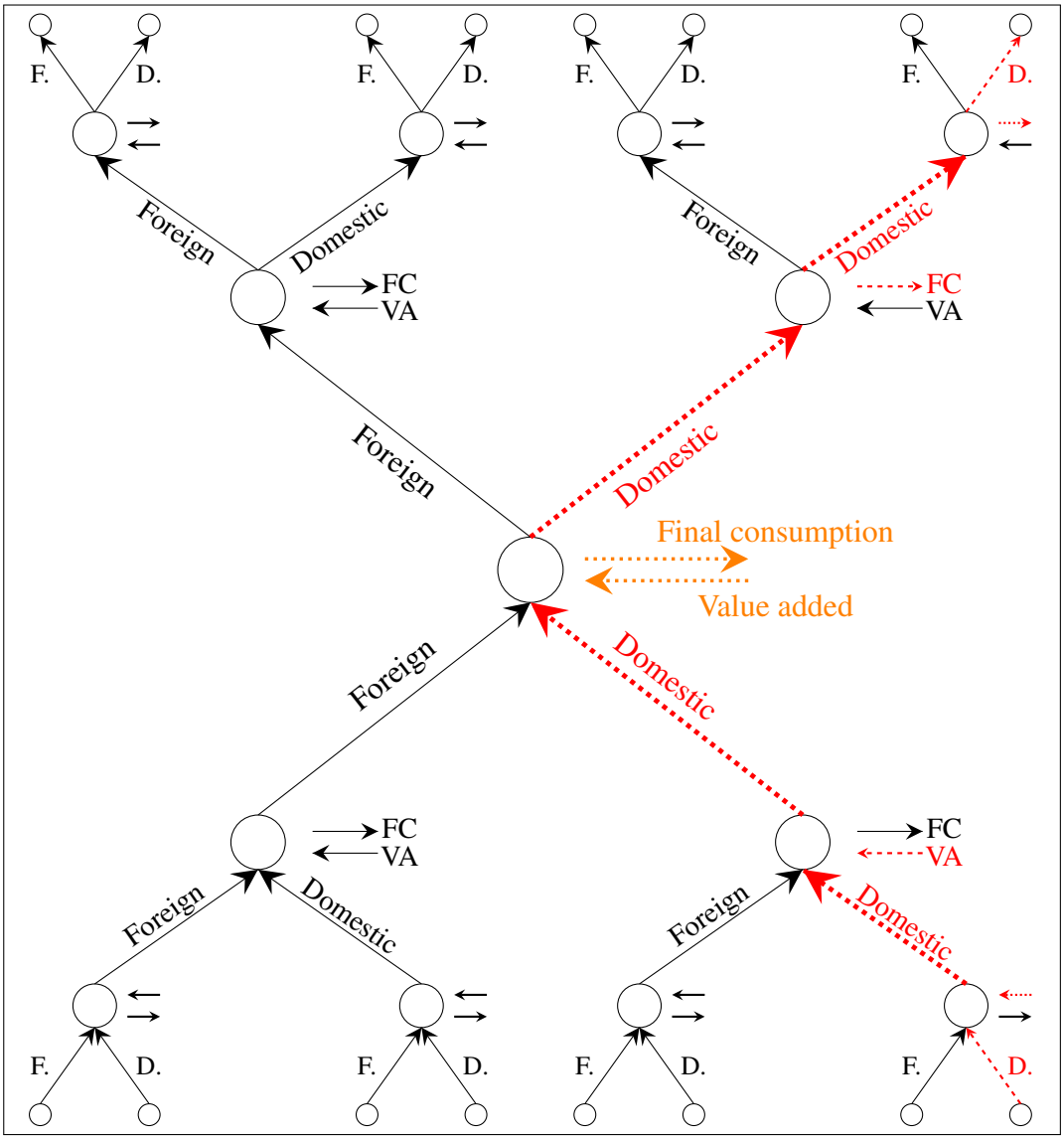


Figure 1: Value chain tree.

Each element of L thus measures the number of vertical transactions of embodied value, which is the value produced in previous stages that is transferred to the production process of further stages. Measuring each sector-country pair L measures the average number of vertical transactions that are required for a value-added to reach final demand, including all the transfers of the embodied value of intermediate inputs. It, therefore, measures the total length of a value chain in which observed sector is only a single stage, measuring vertical distance both below and above it and represents a measure of vertical fragmentation. The smallest total value chain length is equal to 1 and represents a production without any (upstream or downstream) production sharing (household services would be a good example of a sector with almost no substantial intermediate inputs and no upstream production stages). In our disaggregation of production NVCs have an automatic total value chain length (conditional on observing NVCs part) of 1 by construction.

have to be equal to 1. Our interpretation of vertical distance measure thus serves to clarify that useful disaggregation of value chain length when expressed in its downstream and upstream parts separately consists of transactions between firms and unity. Proposed new concept of total value chain length omits transactions covering payments of wages, profits and rents (because of no explanatory value and *a priori* given unity) and includes only downstream and upstream transactions between firms plus a final transaction to consumer, which gives some explanatory power only when disaggregated into transaction to domestic consumer (from point of final production) and exporting to foreign consumers.

3.3.2 Total DVC length

We define total DVC length as total value chain length of DVCs conditional on observing DVCs. To express total DVC length we first need to derive unconditional upstream and downstream DVC lengths, measured as average number of vertical transactions in terms of total output:

$$DVC_L^{Up} = \underbrace{\left\{\left[\mathbf{1}^T(I-A)\right]\left[(I-A_D)^{-1}\right]\right\}^T}_{\text{Share of downstream domestic linkages}} \odot \underbrace{\left\{(I-G_D)^{-1}G_D\mathbf{1}\right\}}_{\text{Number of domestic upstream transactions}}$$

$$DVC_L^{Down} = \underbrace{\left\{\mathbf{1}^T\left[(I-A_D)^{-1}A_D\right]\right\}^T}_{\text{Number of domestic downstream transactions}} \odot \underbrace{\left\{(I-G_D)^{-1}\left[(I-G)\mathbf{1}\right]\right\}}_{\text{Share of upstream domestic linkages}}$$

For unconditional downstream domestic length total value chain length we multiply the widely used expression for measuring number of domestic downstream vertical transactions with share of final stage production of observed sector-country plus share of purely domestic forward linkages. *Vice versa* for unconditional upstream domestic total value chain length we multiply number of upstream domestic vertical transactions with share of observed sector-country value added share plus share of purely domestic downstream linkages. As pointed out, the value chain that is purely domestic in its downstream component can still be part of a GVC when also observed in its upstream extension. Multiplications with expressed shares addresses this and enables observation of downstream and upstream value chain length that belong to DVCs as defined along the whole value chain length.

Because expressed unconditional downstream and upstream value chain lengths are expressed in terms of total output of observed sector-country, we need to express total DVC length measured by vertical transaction in terms of DVC's total output using Hadamard division and adding $\mathbf{1}$ to account for the final transaction to consumer:

$$DVC_L = \underbrace{DVC_L^{Up} \oslash DVCs}_{\text{Upstream DVC length}} + \underbrace{DVC_L^{Down} \oslash DVCs}_{\text{Downstream DVC length}} + \underbrace{\mathbf{1}}_{\text{Transaction to final consumer}}$$

Such formulation expresses the number of vertical transactions along DVCs for each sector-country pair, conditional on DVC observation.

3.3.3 Total GVC length

Similarly to total DVC length, we define total GVC length as total value chain length of GVCs conditional on observing GVCs. To express total GVC length we first derive unconditional upstream and downstream GVC lengths, measured as average number of vertical transactions in terms of total output:

$$GVC_L^{Up} = \underbrace{(I-G)^{-1}G\mathbf{1}}_{\text{Number of upstream transactions}} - \underbrace{\left\{\left[\mathbf{1}^T(I-A)\right]\left[(I-A_D)^{-1}\right]\right\}^T \odot \left\{(I-G_D)^{-1}G_D\mathbf{1}\right\}}_{\text{Number of upstream transactions along domestic value chains}}$$

$$GVC_L^{Down} = \underbrace{\mathbf{1}^T(I-A)^{-1}A}_{\text{Number of downstream transactions}} - \underbrace{\left\{\mathbf{1}^T\left[(I-A_D)^{-1}A_D\right]\right\}^T \odot \left\{(I-G_D)^{-1}\left[(I-G)\mathbf{1}\right]\right\}}_{\text{Number of downstream transactions along domestic value chains}}$$

For both unconditional upstream and downstream GVC length we subtract number of DVC transactions from total value chain transactions, thus taking into account all the value chains that have at least one cross-border transaction between firms, regardless if such a transaction is taking place downstream or upstream

in the value chain. Similarly as with total DVC length, we express total GVC length by expressing total value chain length of GVCs measured by vertical transaction in terms of GVC's share of output:

$$GVC_L = \underbrace{GVC_L^{Up} \oslash GVCs}_{\text{Upstream GVC length}} + \underbrace{GVC_L^{Down} \oslash GVCs}_{\text{Downstream GVC length}} + \underbrace{\mathbf{1}}_{\text{Transaction to final consumer}}$$

Such formulation of GVC_L expresses the number of vertical transactions along the GVCs for each sector-country, conditional on GVC observation.

3.3.4 Total Simple GVC length

Expression of unconditional downstream number of transactions between firms that cross border only once, measured as average number of vertical transactions in terms of total output, consists of 2 parts:
(1.) number of downstream transactions along purely domestic value chains (in downstream part) which in forward linkages cross border once and
(2.) number of downstream transactions along value chains that cross border only once (in downstream part) and continue along forward purely domestic linkages or end as final production stage of observed sector-country pair.

$$SGVC_L^{Down} = \underbrace{\left\{\mathbf{1}^T\left[(I-A_D)^{-1}A_D\right]\right\}^T}_{\text{Number of domestic downstream transactions}} \odot \underbrace{\left\{(I-G_D)^{-1}G_{CB}(I-G_D)^{-1}\left[(I-G)\mathbf{1}\right]\right\}}_{\text{Share of upstream linkages that cross border only once}} +$$

$$+ \underbrace{\left\{\mathbf{1}^T\left[(I-A_D)^{-1}A_{CD}(I-A_D)^{-1}\right]\right\}^T}_{\text{Number of downstream transactions that cross border only once}} \odot \underbrace{\left\{(I-G_D)^{-1}\left[(I-G)\mathbf{1}\right]\right\}}_{\text{Share of upstream domestic linkages}}$$

Similarly expression of unconditional upstream number of transactions between firms that cross border only once consists of:
(1.) number of upstream transactions along domestic value chains (in upstream part) which in downstream linkages cross border exactly once and
(2.) number of upstream transactions along value chains that cross border only once (in upstream part) and have only domestic downstream linkages or start as value added of observed sector-country pair.

$$SGVC_L^{Up} = \underbrace{\left\{\left[\mathbf{1}^T(I-A)\right]\left[(I-A_D)^{-1}A_{CD}(I-A_D)^{-1}\right]\right\}^T}_{\text{Share of upstream linkages that cross border only once}} \odot \underbrace{\left\{(I-G_D)^{-1}G_D\mathbf{1}\right\}}_{\text{Number of domestic upstream transactions}} +$$

$$+ \underbrace{\left\{\left[\mathbf{1}^T(I-A)\right]\left[(I-A_D)^{-1}\right]\right\}^T}_{\text{Share of downstream domestic linkages}} \odot \underbrace{\left\{(I-G_D)^{-1}G_{CD}(I-G_D)^{-1}\mathbf{1}\right\}}_{\text{Number of upstream transactions that cross border only once}}$$

Finally, we express total simple GVC length by expressing total value chain length of simple GVCs measured by vertical transaction in terms of simple GVC's share of output:

$$SGVC_L = \underbrace{SGVC_L^{Up} \oslash SGVCs}_{\text{Upstream SGVC length}} + \underbrace{SGVC_L^{Down} \oslash SGVCs}_{\text{Downstream SGVC length}} + \underbrace{\mathbf{1}}_{\text{Transaction to final consumer}}$$

This expresses the number of vertical transactions along the simple GVCs for each sector-country pair, conditional on simple GVC observation.

3.3.5 Total Complex GVC length

To measure complex GVC length, we first express unconditional downstream and upstream complex GVC length by subtractions of simple GVC transactions from all the GVC transactions for upstream and downstream components separately:

$$CGVC_L^{Up} = SGVC_L^{Up} - GVC_L^{Up}$$

$$CGVC_L^{Down} = SGVC_L^{Down} - GVC_L^{Down}$$

We express total complex GVC length by expressing total value chain length of complex GVCs measured by vertical transaction in terms of complex GVC's share of output:

$$CGVC_L = \underbrace{CGVC_L^{Up} \oslash CGVCs}_{\text{Upstream CGVC length}} + \underbrace{CGVC_L^{Down} \oslash CGVCs}_{\text{Downstream CGVC length}} + \underbrace{\mathbf{1}}_{\text{Transaction to final consumer}}$$

Total complex GVC length in the final expression therefore measures the number of vertical transactions along the complex GVCs for each sector-country, conditional on complex GVC observation.

3.4 Relative position in value chains

While total value chain length is a good measure of vertical fragmentation of the value chain as a whole (or each of its parts), it does not give any insight on the position of the observed sector in value chain. In line with previous decompositions we define relative upstream (R^{Up}) and relative downstream (R^{Down}) position measure:

$$R^{Up} = \underbrace{[\mathbf{1}^T(I-A)^{-1}A]^T}_{\text{Downstream transactions between firms}} \oslash \underbrace{\left\{[\mathbf{1}^T(I-A)^{-1}A]^T + [(I-G)^{-1}G]\mathbf{1}\right\}}_{\text{All transactions between firms}}$$

$$R^{Down} = \underbrace{[(I-G)^{-1}G]\mathbf{1}}_{\text{Uptream transactions between firms}} \oslash \underbrace{\left\{[\mathbf{1}^T(I-A)^{-1}A]^T + [(I-G)^{-1}G]\mathbf{1}\right\}}_{\text{All transactions between firms}}$$

Measures are defined symmetrically and are inversely related, therefore the second relative position measure does not provide any additional information, since $R^{Down} = \mathbf{1} - R^{Up}$. Both measures are bounded between 0 and 1, where higher upstream relative position values R^{Up} represent a higher relative upstream position in the value chain (relatively longer downstream length compared to the shorter distance to final demand). Lower values show the opposite (relatively longer distance to final demand compared to shorter downstream length). Relative position measure represents the share of the number of either upstream or downstream vertical transactions between firms among all of the vertical transactions between firms, informing about the downstream and upstream shares in total value chain length of each sector-country pair, R^{Up} representing downstream share and $1 - R^{Up}$ representing upstream share and *vice versa*.

The construction of relative position measures for sector-country is applicable for the analysis of diverse value chains - domestic value chains and global value chains, as well as for simple and complex GVC:

$$R_{DVC}^{Up} = DVC_L^{Down} \oslash (DVC_L^{Up} + DVC_L^{Down})$$
$$R_{GVC}^{Up} = GVC_L^{Down} \oslash (GVC_L^{Up} + GVC_L^{Down})$$
$$R_{SGVC}^{Up} = SGVC_L^{Down} \oslash (SGVC_L^{Up} + SGVC_L^{Down})$$
$$R_{CGVC}^{Up} = CGVC_L^{Down} \oslash (CGVC_L^{Up} + CGVC_L^{Down})$$

3.5 Decomposition of transaction to final consumer

To complete proposed decomposition of total output (participation shares) as well as measures of total value chain length and relative position, we also propose decomposition based on the final transaction to consumers, which could be domestic transaction or export to foreign consumers (from the perspective of country of final production stage). We use the matrix F that includes all the information about final consumption and construct a matrix of coefficients E representing shares in total final consumption $E = F \oslash (F\vec{1})$. We decompose a matrix E in the same manner as matrices A and G, the only difference being the dimension of blocks, which are in our case m dimensional vectors (describing final consumption for each sector), the block structure still $n \times n$ dimensional block matrix, where diagonal block vectors represent domestic final consumption shares and off-diagonal block vectors foreign final consumption shares $E = E_{CB} + E_D$.

Since by construction $E\vec{1} = \mathbf{1}$ and thus $E_D\vec{1} + E_{CB}\vec{1} = \mathbf{1}$, we can substitute the vector $\mathbf{1}$ in all the $(I - G)\mathbf{1}$, $(I - G)^{-1}G\mathbf{1}$ or $(I - G_D)^{-1}G_D\mathbf{1}$ parts of our derivation of proposed measures with our decomposition and with this easily desegment all presented measures into two - first referring to value chain part that ends with domestic consumption and second that ends with export to final consumers. Finally, Table 2 systematically describes newly proposed measures for various types of value chain.

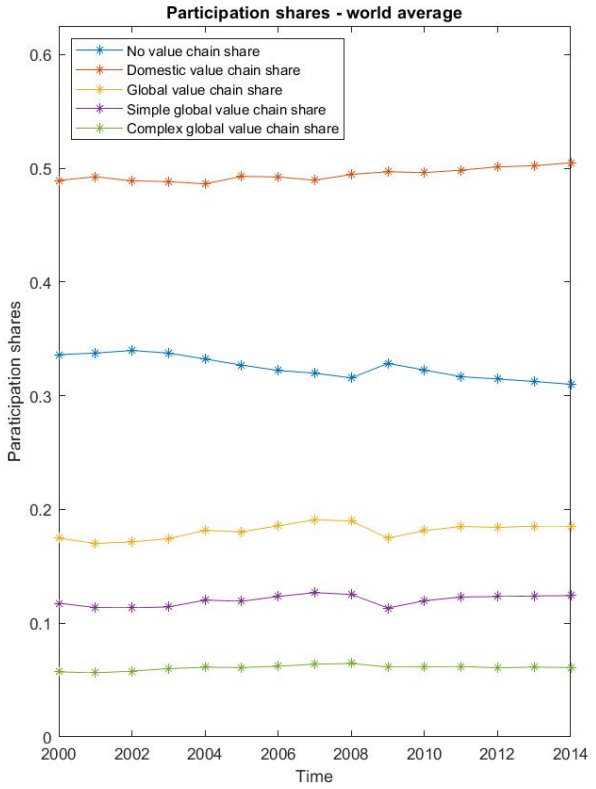
Object	Participation shares	Length	Relative position
No value chain	No value chain with domestic final consumption share	1	/
	No value chain with final exporting share	1	/
Domestic value chain	Domestic value chain with domestic final consumption share	Length of domestic value chain with final domestic consumption	Relative position in domestic value chain with final domestic consumption
	Domestic value chain with final exporting share	Length of domestic value chain with final export	Relative position in domestic value chain with final export
Global value chain	Global value chain with domestic final consumption share	Length of global value chain with domestic final consumption	Relative position in global value chain with final consumption
	Global value chain with final exporting share	Length of global value chain with final exporting	Relative position in global value chain with final exporting
Simple global value chain	Simple global value chain with domestic final consumption share	Length of simple global value chain with domestic final consumption	Relative position in simple global value chain with domestic final consumption
	Simple global value chain with final exporting share	Length of simple global value chain with final exporting	Relative position in simple global value chain with final exporting
Complex global value chain	Complex global value chain with domestic final consumption share	Length of complex global value chain with domestic final consumption	Relative position in complex global value chains with domestic final consumption
	Complex global value chain with final exporting share	Length of complex global value chain with final exporting	Relative position in complex global value chains with final export

Table 2: Proposed measures.

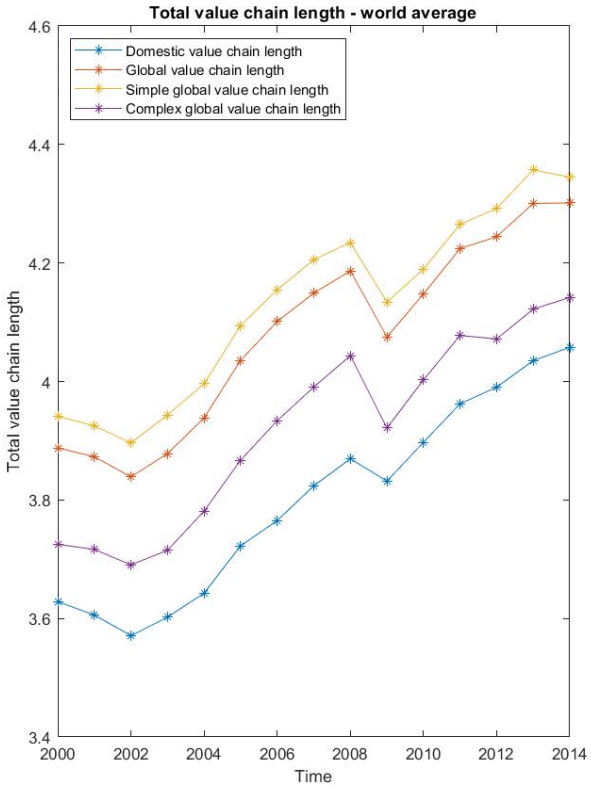
4 Empirical demonstration of new measures

Proposed measures increase possibilities for empirical application and static analysis of international production and trade. On the one hand improved disaggregation of domestic component is first to enable thorough domestic value chain analysis to complement GVC analysis and explore structural changes in the economies as they become increasingly integrated, on the other hand conceptually new methodology of GVC participation shares and value chain lengths gives better insight into basic international economic proportions. Disaggregation of all value chains on those that end with exporting or consumption in the last country of production, enables to compare relative positions, total value chain lengths as well as participation shares with respect to this criterion, which enables testing of fundamental theoretical premises

of international trade and production theory separately on different value chain objects (DVCs, simple or complex GVCs), countries and sectors.



(a) *Figure 2: Participation shares - world average.*



(b) *Figure 3: Total value chain length - world average.*

Source: WIOD, 2016; own calculations.

Due to limitations of this paper and its primarily methodological focus we demonstrate only some very basic empirical results. We show world averages of participation rates and value chain lengths based on WIOD 2016 data. Using our methodological approach we observe that world average GVC share of output is consistently above 17%, reached almost 20% at the peak before the global recession and then stagnated slightly bellow that value until 2014 (Figure 2). Our expectations that existing participation shares of GVCs are undervalued are thus confirmed, since contemporary GVC reports demonstrate GVC component ranging between 10% and 15% (GVC report, 2017, 2; 2019, 12). In contrast to existing disaggregation approach, which shows increase in GVCs participation at the expense of domestic component (GVC report, 2017, 2; 2019, 12), our methodology shows stable and even increasing share of DVCs, while the major drop occurs with respect to the share of output that has no value chain (Figure 2). Another interesting observation made on simple world average participation share disaggregation is bigger stability of complex GVCs compared to simple GVCs - global recession had a major effect on the latter but only slight on the former.

With respect to total value chain length (Figure 3) we firstly note that an average increase in total value chain length is in observed timespan (2000-2014) larger than average differences between observed categories: on average shortest value chains - domestic value chains – were in 2014 on average longer than global value chains of 2000. The dynamic pattern of total value chain lengthening is consistent in the whole period with the exception of periods 2000-2002 and 2008-2009 when we observe shortening of value chains. Interesting observation is also fairly consistent constant difference between average DVC, GVC, SGVC and CGVC length through the whole time span. The dynamics slightly before and after

the global recession show that lengthening of value chains was more pronounced for complex and simple GVCs before the crisis compared to DVCs, but those were also shortened at the time of crisis much more than DVCs, which appear least effected by the crisis of all value chains. With our measure we also observe on average longer simple GVCs compared to complex GVCs, simply meaning that global value chains that include production sharing of only two countries are on average longer than the rest of the global value chains.

5 Conclusion

In the article we propose new methodology for measuring participation shares, total value chain length and relative position within different types of value chains. We present a more comprehensive conceptualisation and more appropriate derivation of proposed measures. With this we solve conceptual inconsistencies in existing measures, primarily those caused by isolated disaggregation of upstream and downstream parts of value chains, which disregard their connection. Apart from that, we offer researchers new possibilities to conduct analyses on different levels of disaggregation, be it comparative geographical analysis (e.g. between two countries or between groups of countries) or observing value chain dynamics in different sectoral disaggregation. Our methodology provides a ground for improved and corrected insight into different types of value chains as well as broader tool-kit, useful for various extensions of research.

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